

Necessary Conditions for Internal Models with regard to Annuitant Mortality.

1. Summary.

1.1 This paper considers what an approach to modelling mortality variation for purposes of Solvency 2 should cover.

1.2 I examine the three main components of annuitant mortality namely

- 1) Binomial variation of mortality within the portfolio
- 2) Annual fluctuation of the background population mortality
- 3) Trend Variation where the rate of mortality improves at a varying rate for the whole population

1.3 Throughout I refer to annuity payments from a life insurance company but identical considerations apply to pension funds paying pensions to retired members.

1.4 Much of the material from this paper is drawn from work presented to the Society of Actuaries in Ireland in October 2004, Jeffery & Olivier (1) (referred to as J&O). This work was prepared following enquiry from the Irish Pensions Board as to the impact of mortality variation on reserves. Due to the relatively small numbers of lives in Ireland, data used for the study was based on England & Wales population data.

2. Binomial Fluctuation

2.1 For each annuitant there is a chance that he/she will die each year and so the stream of payments and its value are uncertain. It is relatively simple to model this using random number generators to model that person as dying if the generated random number (between 0 and 1) is less than or equal to q_x . This can then be done for each live in the portfolio and be rolled forward until all lives are extinguished. The flow of payments can be discounted back to the present. By doing this for a large number of stochastic runs and ranking the discounted values, confidence limits at any desired probability level can be obtained.

2.2 By way of example this has been done for an nominal portfolio of identical lives assumed to be all male age 60 exact and assuming underlying mortality of the “medium cohort projection” produced by the Institute of Actuaries in England and Wales. An interest rate of 4% was assumed. The following results were obtained.

Number of Lives	Extra reserve for binomial variation, expressed as percentage of the basic reserve		
	95th percentile	99th percentile	99.5th percentile
10	11.75%	15.25%	16.37%
100	3.98%	5.46%	6.06%
1000	1.27%	1.81%	2.02%
10,000	0.41%	0.58%	0.64%
100,000	0.13%	0.19%	0.21%
1,000,000	0.04%	0.06%	0.06%

2.3 These percentages are the extra reserve (or “required margin”) that is necessary to be 95% (or 99% or 99.5%) certain that sufficient reserve is held, expressed as a percentage of the expected reserve.

2.4 One feature noted during this modelling was that a very large number stochastic runs was necessary before these figures became stable if the lives were 100 or less. 5000 iterations were not sufficient.

2.5 As can be seen and as would be expected the results follow a “root n” pattern, that is the required margin for stochastic fluctuation increases in absolute terms in proportion to the number of lives so proportionately the margin required for stochastic variation falls as a percentage of the absolute reserve. This is a diversifiable risk as it can be reduced by the undertaking of extra risks.

2.6 Other features of this variation are as follows:-

- The required margin decreases with increases in interest rates (this contrasts with assurances where the margin rises with increases in interest rates)
- The required margin rises with increasing age, if expressed as a percentage of basic reserves. The absolute margin appears to be relatively stable.
- The required margin varies according to mortality table chosen but as a very rough rule is proportional to $q_x(1-q_x)$ at the current age. The subsequent shape does have impact on the required margin so this is only very approximate.

2.7 Given that the required margin at the 99% level is only 0.21% for 100,000 lives, it might be wondered whether this aspect needs modelling at all. However portfolios of identical risks are rare, a more usual situation is that the annuity varies from person to person. This can have a very significant effect.

2.8 This has been demonstrated in Richards & Jones (2) where (in Table 9, page 35) they calculated the required margin on some notional portfolios as follows

Scheme benefit Structure	Totals:		Interest Rate	
	Lives	Benefits	2.5%	5.0%
50 lives receiving 1 p.a.	50	50 p.a.	10.3%	8.4%
41x1 p.a. 1x5 p.a. 8x0.5 p.a.	50	50 p.a.	11.9%	9.5%
31x1 p.a. 1x10 p.a. 18x0.5 p.a.	50	50 p.a.	15.2%	11.6%

2.9 This table shows that even very moderate variation of amount payable causes significant increases in required margin

2.8 Another demonstration of this point is given in J&O (1) where we considered a hypothetical pension scheme consisting of one manager and a (varying) number of staff. The manager would be paid a multiple of the basic staff annuity but otherwise would be identical in age in mortality.

Extra reserve for binomial variation, expressed as percentage of the basic reserve					
Multiple of benefit	Number of staff				
	10	50	100	500	1000
No manager	15.2%	7.5%	5.5%	2.4%	1.8%
3	50.2%	14.5%	8.5%	3.1%	2.1%
5	74.7%	19.2%	10.9%	3.4%	2.4%
10	138.5%	30.9%	16.9%	4.7%	2.9%

2.9 As can be seen the impact of the manager is very significant on the required margin. Even if there are 1000 other staff, a multiple of 3 times basic annuity increases the required margin by one third.

2.10 To examine this for an actual portfolio of annuities in payment we were kindly supplied with data by Friends First, one of the life companies writing annuity business in the Irish market. The portfolio consisted of 3683 lives (note that the Irish market is small due to the relatively young population). We carried out similar stochastic analysis as detailed above on the actual portfolio and a comparison portfolio of 3683 identical lives

	Extra reserve for binomial variation, expressed as percentage of the basic reserve		
	95th percentile	99th percentile	99.5th percentile
Friends First Portfolio	2.05%	2.90%	3.22%
Comparison portfolio	0.66%	0.84%	1.05%

2.11 The difference is very substantial and demonstrates the need to model portfolio variation explicitly.

2.12 The question arises as to whether a portfolio of annuities can be diversified to the point of needing such a low required margin for Binomial variation that it can safely be ignored. The work above work suggest that this would be the case if the annuity amounts were identical but this is a comparatively rare instance. If we consider that annuity amounts are related to wealth and that wealth is generally held to follow Pareto's law so that there are always outliers of high amounts then it seems unlikely that binomial variation can be diversified away for these outliers. It is therefore clear that modelling of this risk should take place and should be required in Solvency 2 models.

2.13 There are practical considerations that can make modelling difficult. They should be remedied if possible.

- Individuals may have more than one policy
- Reversionary benefits may exist.

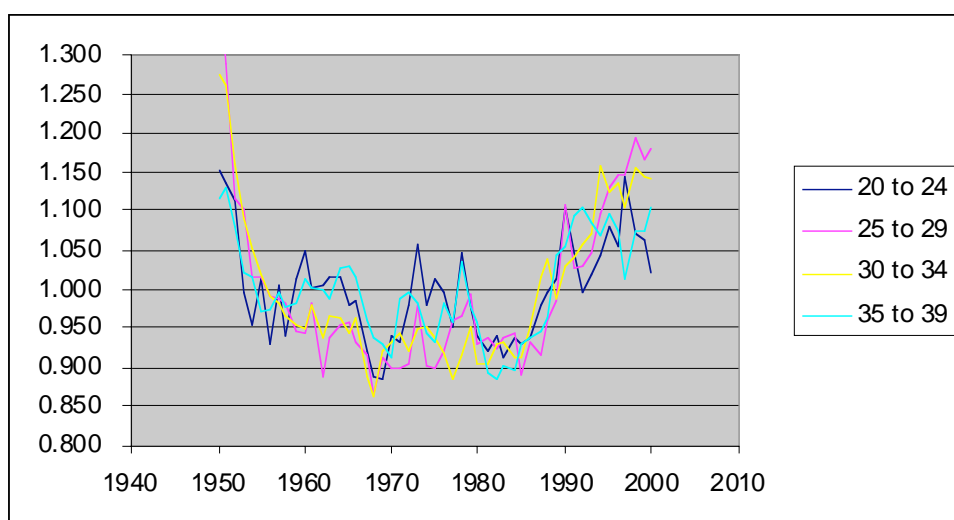
2.14 There are effects that modelling cannot pick up very easily

- For most companies the annuitant lives are unlikely to be completely independent of each other. Many husband and wives will be insured with the same company through widow and widower benefit arrangements. If the liability arises from an occupational pension scheme there will be correlation between lives from geographical location and from trade. Many sales sources for financial services exploit affinity groups and particular locations. However the risk that exists for insurance risk from accumulation is more significant than it is for annuity where survival is the risk factor rather than death
- Correlation between wealth and longevity are generally thought to exist. This could imply that larger annuity amounts would have better survival prospects. Certainly “amounts” experiences generally show lower mortality than “lives”. Beyond using amounts experience as base table, we would suggest that trying to model this in the binomial variation is too complex at present.

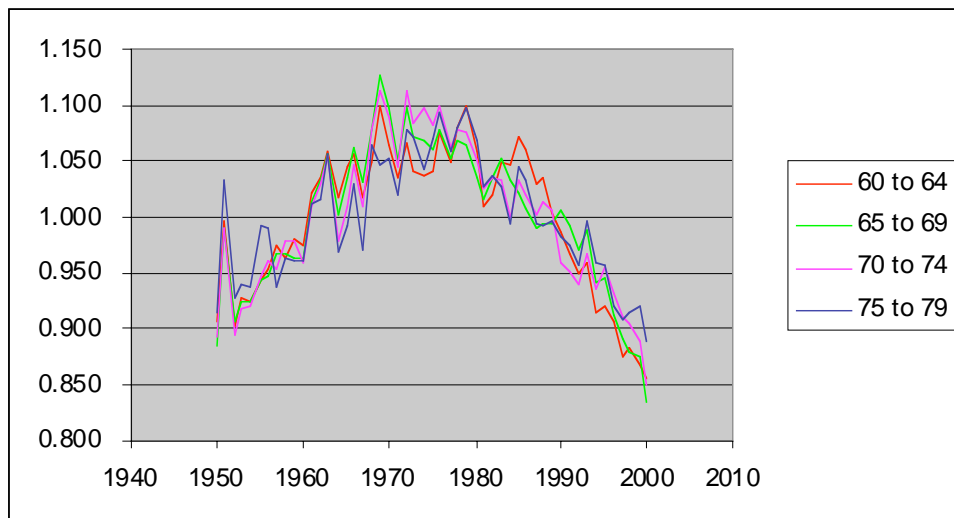
3. Population Fluctuations

3.1 Population mortality itself varies from year to year by far more than stochastic variation would imply. Reasons for this include climatic variation and epidemics (notably flu). There is demographic research which shows that (for the UK) there are more deaths in the winter than at other times (excess winter mortality or EWM). This EWM is correlated to the average temperature of the winter but is not purely due to hypothermia. There is also strong correlation to flu epidemics though the excess deaths are not all recorded as from influenza. Lastly it should be noted that EWM for the UK is higher than comparable countries. Details may be found in Curwen (3).

3.2 Another way of demonstrating this is to compare year on year crude mortality rates for different ages. The following graphs are the trend adjusted rates for 1950 to 2000 for England and Wales. The table for younger ages show only a loose correlation between year on year patterns of mortality.



3.3 The graphs for older ages by contrast show very closely linked behaviour. All age groups do well or suffer at the same time. This is clear evidence of population fluctuation.



3.4 In considering the potential impact of such effects it is essential to understand the cumulative effect. There are three possible effects. Firstly a bad year (with a high mortality rate) might be the bringing forward of a number of deaths so that a mean reverting behaviour would occur. This would mean that population fluctuations would be of very little long term impact on liability. Life company liabilities would be increased by one good year to fall the following year. I have named this the “herd” effect based on the concept of deaths of weaker members of the collective herd leaving the totality stronger.

3.5 Alternatively the opposite might occur. A bad year may affect us all, killing a few but leaving even the survivors weaker. This would mean that cumulative effects of successive bad or good years would be greater still. I have called this the vitality effect.

3.6 The last hypothesis which I have called the independence effect would be that the mortality in one year is unaffected by whether previous years display higher or lower than average mortality.

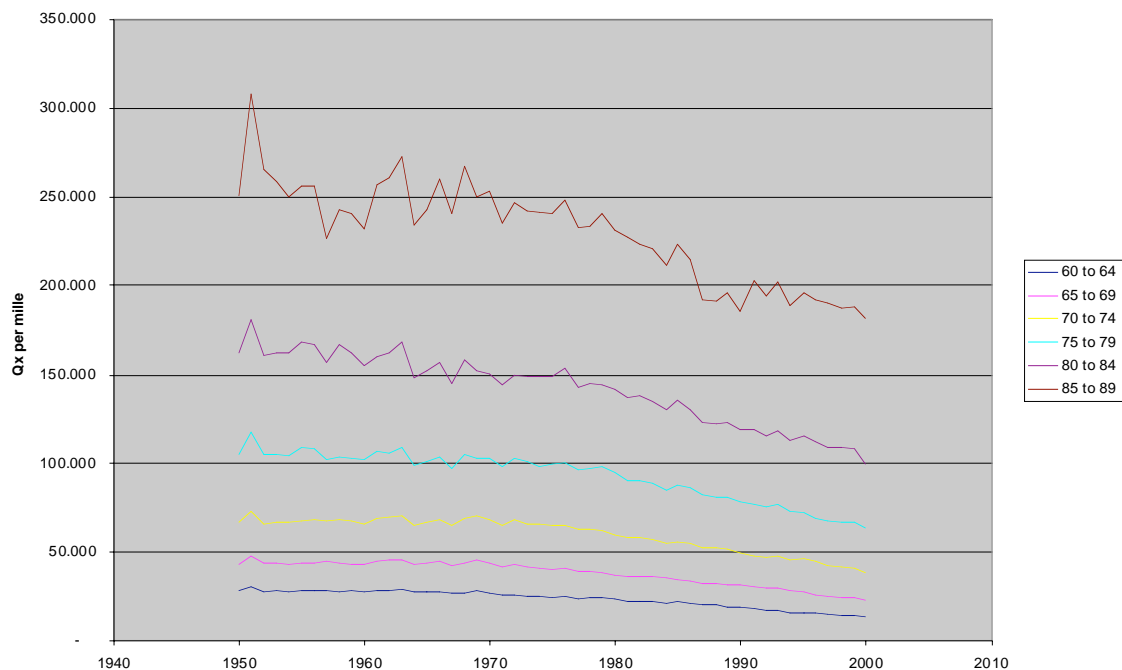
3.7 Clearly adverse years for the annuitants are beneficial for the life company as death will terminate liability.

3.8 It seems reasonable to suppose that whether herd, vitality or independence behaviour exists could depend on cause of death and on country. To examine this I suggest that mortality is compared in the years before and after a peak or trough year. For example if year N is a peak year, then mortality in year $(N+1)$ would be lower than in year $(N-1)$ if the herd effect applied and higher if the vitality effect applied. Similar arguments apply for trough years in mortality.

3.9 J&O contains detail of analysis carried out on England and Wales mortality which compares $(N+1)$ and $(N-1)$ levels (after allowing for trend). This showed no pattern

with half of the peak mortality years and half of the trough years suggesting herd effects and half suggesting vitality effects. This leads to the tentative conclusion that one year is not much affected by another (in England and Wales at least).

3.10 Examination of the England and Wales experience also reveals an interesting pattern. Not only was the rate of improvement was sharply greater in the period 1980 to 2000 than the period 1950 to 1980 but the variation from year to year is strikingly less also. This can be seen on a graph of the crude mortality rates as follows.



3.11 To show this statistically the table below gives the comparative variation for pre and post 1980 period and the least squares trend (average rate of improvement).

Age Band	Comparative Variation		Trend	
	Pre 1980	Post 1980	Pre 1980	Post 1980
60 to 64	17.4%	11.5%	-0.172	-0.504
65 to 69	20.2%	8.6%	-0.191	-0.718
70 to 74	18.5%	7.3%	-0.175	-1.004
75 to 79	16.9%	7.3%	-0.369	-1.429
80 to 84	18.3%	9.0%	-0.847	-1.802
85 to 89	31.0%	17.4%	-0.843	-2.036

3.12 This suggests that for England and Wales the annual fluctuation has reduced since 1980 and that this has coincided with an increase in the rate of improvement.

3.13 With only 20 years experience to examine it is hard to draw conclusions but it would not be unreasonable to model this as a normal distribution of a percentage

increase or reduction in annual mortality applied to all ages, with standard deviation 1.7%

4. Trend

4.1 There is no doubt that, for a large company, trend variation is the most important. The pace of improvements in recent years has exceeded previous projections and is causing widespread social change.

4.2 There is a “head of steam” behind such change. That is we know that some of the factors that have led to past change are still contributing to future change. These factors include changes in smoking behaviour and improved cardiovascular medicines. Possible others may be increasing wealth and flu vaccines.

4.3 There is also the well known cohort effect. Recent projections in the UK Richards, Kirkby & Currie (4) suggest a projected improvement for age 70 of 3.5% p.a. over the next 40 years. It is important not to double count this with the medical advances referred to above as it is these advances that have created the cohort improvement.

4.3 This suggests that two possible approaches to trend modelling exist. We can undertake analysis of the current medical environment, project that forward and then use that as our baseline for stochastic analysis. Alternatively we can use the current rate of improvement as the starting point and project trend changes about that level. An approach based on this might take the recent rate of improvement as the expected improvement going forward.

4.4 Using the latter method poses the lesser burden as all that is needed is a knowledge of recent improvement rates. However I suggest that by avoiding examination of expected improvements already due from existing medical improvements we risk underestimating improvement at present. Projections have been prepared e.g. in Richards, Kirkby & Currie (4) and one such as these should be the starting “neutral” point for trend projection.

4.5 There is a great deal of money being spent on medical research with the aim of improving longevity. It is reasonable to assume that some of this will be successful. If the starting projection is based only on the known medical advances then the trend factor should be assumed to have a positive mean. However if the starting projection is based on cohort projection methods then some medical improvement may already be implicitly allowed for.

4.6 It has been a feature recently that socio economic class mortality differentials have widened.. This has meant that some companies have experienced improvements at greater rates than those experienced by population mortality, particularly when experience is assessed on liability rather than numbers of lives. Therefore in assessing the distribution of the trend population the parameters should allow for higher improvements than have been shown by the corresponding population.

4.7 If a CTE (or “tailvar”) approach is to be used then the whole population of possible improvements needs to be assessed. This should include a very substantial improvement that could occur from concatenation of medical advances inter-reacting

in a benign manner. This is best illustrated by the example of AIDS when a number of drug treatments have lead to major reductions in mortality when used in combination.

5. Summary of Recommendations

- A. That each company be required to investigate its own exposure to stochastic fluctuation.
- B. This exposure should allow for the portfolio features and not be based purely on numbers of lives
- C. Large numbers of iterations may be necessary to produce stability in outcomes.
- D. Population variation should be examined for each geographical area to determine whether herd or vitality effects exist, and to assess the distribution of annual fluctuations
- E. Trend variation should be based on a projection of future improvements that allows for known medical factors whose effects are not fully developed.
- F. Trend should assume that the mean of the future development will be reductions in mortality and this should be assumed either explicitly or implicitly
- G. Annuitants should be assumed to enjoy better improvements to mortality than the population unless there is evidence to the contrary.

6. References

- (1) Jeffery & Olivier “Stochastic Mortality Models” Presentation to Society of Actuaries in Ireland 20th October 2004
- (2) Richards & Jones “Financial Aspects of Longevity Risk” presented to Staple Inn Actuarial Society 26th October 2004
- (3) Curwen M. “Excess winter mortality in England and Wales with special reference to the effects of temperature and influenza” published within “The Health of Adult Britain 1841-1994” published UK Office for National Statistics”.
- (4) Richards, Kirkby & Currie “The importance of year of birth in two dimensional mortality data” presented to the Institute of Actuaries 24 October 2005.