ON CALCULATION of SURPLUS VALUE USING STOCHASTIC MODELLING

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Abstract

Embedded value (EV) and one of its components - Surplus value (SV) - is one of the most important figures when estimating future profits of life insurance company. Usually it is assumed that processes of life insurance company are governed by multiple state homogeneous Markov process, however this is not always true in practice. In the paper we deal with the case when second order Markov chain is present at some parts of multiple state model. We present some shortcomings of deterministic Surplus value and propose stochastic model for calculation of Surplus value. While much attention now is paid to stochastic modelling of economic assumptions, such as interest rate, we turn our attention to stochastic modelling of mortality and morbidity. We use Monte Carlo method as basis for developing mortality and morbidity scenarios. Example of specimen calculations is given.

Keywords: Embedded value, Markov chain, Monte Carlo method, Stochastic simulation, Surplus value.

1 Introduction

Today statutory results of insurance company are the only measure of company's performance in Lithuania as well as in many other countries. However it is well known that statutory results alone are not very good measure of company's performance. This is especially important for life insurance companies since assets and liabilities of life insurance companies are of long term nature.

For management purposes it is necessary to obtain more detailed information, especially concerning future profits, which depend on future cash flows. Much information about future profits may be obtained from so-called Embedded value of the company. Embedded value represents value of the company taking into account only "in force" business, that is, current portfolio without any assumptions about future sales. It is obvious that Embedded value will depend highly on model used and conditions assumed, therefore if one wants to use Embedded value as comparison measure of different insurance companies it necessary to use the same model and assumptions. However for management purposes it is important to reflect all processes involved in business as much accurate as it is possible.

The Embedded value is the sum of so-called Present value of future profits (PVFP) and so-called Adjusted net asset value (ANAV):

$$EV = PVFP + ANAV.$$

First term is also called Surplus value (SV), while ANAV consist of free capital or capital tied up in the run-off of the business, after consideration of the taxes to be paid by the company (see [8]).

We will concentrate our attention on calculation of Surplus value (present value of future profits) since our aim is to discuss modelling of some processes observed at life insurance company.

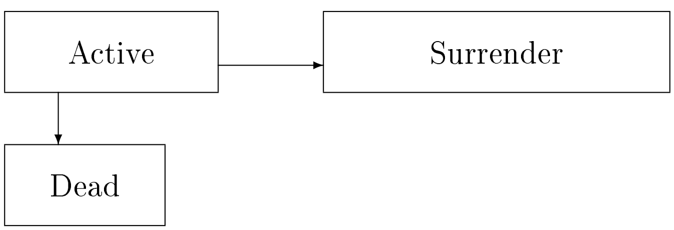
Our paper is organized as follows. In section 2 we present multiple state model used for calculation of Surplus value, shortcomings of deterministic calculations and main features of stochastic modelling of mortality and morbidity. in section 3 we present illustrative results of stochastic calculations. In section 4 we provide recommendations regarding further use of stochastic modelling techniques.

Data used for specimen calculations are taken from Lithuanian market with slight modifications since all figures are used for illustration purposes only.

2 Methodology

Let us assume a single life insurance policy. We will concentrate on unit linked life insurance, however most ideas may be also applied to other types of life insurance. Probably, the simplest way to model possible outcomes of life insurance policy is to use discrete time Markov chain. It assumed that at every time moment t_i every policy may be in one of three states: "Active", "Surrender" or "Death" (see Figure 1).

Figure 1. Multiple state model used for calculation of Surplus value



At the moment t_{i+1} policy may remain in active state with probability p_x or move to states "Dead" or "Surrender" with probabilities q_x^d or q_x^w correspondingly. Transition probabilities usually depend on age (and sex) of policyholder, therefore, we use argument x. States "Dead" and "Surrender" are absorbing states in this model.

While policy stays in "Active" state, premiums P_t are paid to life insurance company at the start of each period t. When policy moves to state "Surrender" or "Death", surrender value S_t expressed as lump sum (or death benefit D_t) is paid to policyholder at the end of period t. When policy expires at time n insurance benefit, equal to reserve V_n, is paid upon termination.

This model may contain additional states, however, the main feature of this and analogous models is Markov property.

Formally we define Markov property for a discrete time Markov chain as follows. Let us assume that state space is a finite subset of $\mathbb{N}$ and $\xi(t)$ is state of chain at time t. Then for any $t, j \in \mathbb{N}$, for any non - negative integers $0 \leq t_1 < t_2 < \cdots < t_k < t$ and for any $i_1, i_2, \ldots i_k \in \mathbb{N}$ the following equation holds:

$$\begin{gathered} P(\xi(t) = j | \xi(t_1) = i_1, \xi(t_2) = i_2, \ldots, \xi(t_k) = i_k) = \\ P(\xi(t) = j | \xi(t_k) = i_k) = q_{i_k j}(t_k, t). \end{gathered} \tag{2.1}$$

The profit (loss), generated by single policy during period t is calculated by the formula:

$$\begin{aligned} PL_t = \quad & P_t + I_t - p_{x+t-1} \Delta V_t - q^d_{x+t-1} DS - \\ & -q^w_{x+t-1} S_t - E_t, \end{aligned} \tag{2.2}$$

where:

PL_t - profit (loss) during period t;
P_t - insurance premiums received at the start of period t;
I_t - investment income during period t;
ΔV_t - changes in reserves during period t,
DS - sum insured;
S_t - surrender value;
E_t - expenses of insurance company incurred during period t;
q^d_x - death probability for a person aged x (depend also on sex of person);
q^w_{x+t-1} - surrender probability for a person aged x;
p_x survival probability for a person aged x.

Using above mentioned model and equation (2.2) as a basis, calculations of profits during periods $t = 1, 2, \ldots$ are made for every policy. The Present value of future profits (PVFP), or Surplus value, of any single policy is calculated by well known formula:

$$PVFP = \sum_{t=1}^{T} \frac{PL_t \times {}_{t-1}p_x}{(1 + rdr)^t}, \tag{2.3}$$

where
T - time left till the end of policy period;
rdr - risk discount rate.

Risk discount rate may vary for different products or different time periods. Though various parameters used in equations (2.2) and (2.3) are usually referred as annual (e.g., mortality and survival probabilities, risk discount rate), above mentioned formulas with very slight modifications may be used for other periods, such as months. We perform calculations on a monthly basis in practice.

Since calculations in (2.2) and (2.3) produce "expected values" using "best estimates" assumptions (or conservative estimates if "best estimates are not available) and no random variables are involved, calculated surplus value is called deterministic surplus value. It is relatively easy to implement and understand deterministic surplus value. Deterministic SV is useful for evaluation of future profits of insurance company and for decisions whether or not profits are on appropriate level. Overall risks are reflected in the discount rate, using higher discount rate for riskier products.

However limiting calculation of surplus value only to deterministic case has some shortcomings.

It is well known that almost all parameters which influence "behavior" of insurance policy, such as mortality and surrender rates, investments return and others, are random, so in practice there arise fluctuations from expected values of number of deaths, surrenders, investment return etc. "Best estimate" is not the only possible scenario. Some risks, such as mortality, may be diversified over the mass of independent policies, however, there still remain a risk that real outcome of mortality losses will differ greatly from its expected value due to huge unexpected losses, such as epidemics, terror attacks like September 11, 2001, and similar. So, with deterministic SV it is impossible to evaluate profits at tails, especially left tail which means low profits.

With deterministic SV it is also impossible to estimate influence of some reinsurance programs (for example, Excess of loss reinsurance) since "best estimate" may become almost best case scenario for reinsurer as according "best estimate" scenario reinsurer will pay no claims at all (see [2]).

Another shortcoming of deterministic SV is that it is not suitable for measurement of interaction of risks. This point is very important due to the fact that sometimes risks which are generally treated as uncorrelated may become dependent under extreme circumstances. The "classical" example of such risks is above mentioned sadly famous events of September 11th, 2001, when aviation, property and life risks became correlated. Such phenomenon is called "tail dependency", under which the extremes of distributions are correlated but their main ranges not (see [6]).

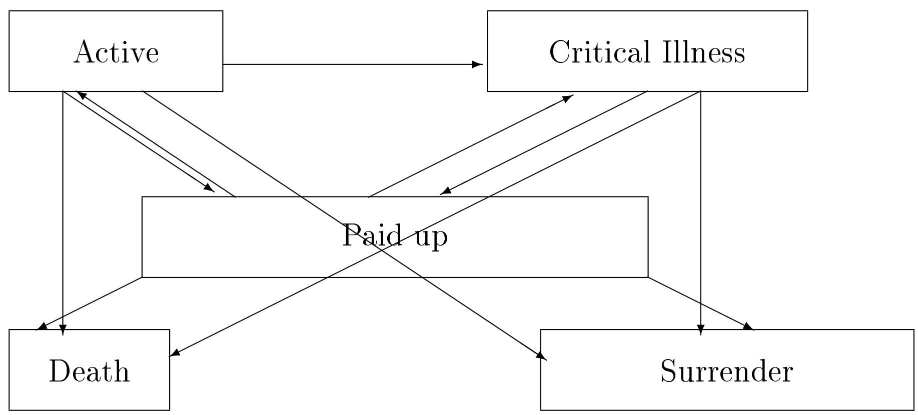
Another example of so-called "tail dependency" is the rate of investment and number of surrenders in unit linked life insurance. In general we do not observe correlation between decrease in rate of return and number of surrenders, however under some specific circumstances those two quantities may become dependent. Normally, we observe surrender rates in the range of 1% - 3% per year. However in the period of 2002 - 2004 due to the policy of Lithuanian government (on February 2, 2002 fixed exchange rate of Lithuanian local currency Litas against the EUR was set and so fixed exchange rate of Litas against USD was cancelled) exchange rate of USD against LTL decreased significantly. Unexpected decrease of USD exchange rate against LTL was the main reason that policyholders who invested their premiums into USD bonds cancelled their policies more often than those not investing into USD bonds. Below we compare surrender rates of policyholders investing into Fund of USD bonds and Fund of World shares. Both experienced fall in interest rates during period 2002 - 2004, however surrender rates among those investing in Fund of USD bonds increased significantly, while among those investing in Fund or World shares remained almost at the same level (see Table 1 and Figure 3 below).

Most probably increase in surrender rate was caused not only by decrease in investment rates but more by the fact that Lithuanian citizens did not expect fall in exchange rate of hard foreign USD against local currency Litas, while decrease in shares prices was not accepted as very unlikely. Above described problem shows that it is quite difficult to predict behavior of policyholders especially when circumstances became extreme (such as sudden fall in prices of US dollar) as behavior of policyholders is influenced not only by economical factors but by psychological factors too. It means that it is very difficult if not impossible to establish correct relationships among "tail dependent" risks.

One more reason for using stochastic versus deterministic surplus value is that processes involved in life insurance company are usually much more complex than that showed in Figure 1. For example, modern life insurance policies involve many additional benefits, such as insurance of Critical Illness or Accidental dismemberment. Policyholder very often has the right to pay up policy. Paid up option means that policyholder chooses not to pay premiums for some time. This option changes significantly cash flows of insurance company and it is usual practice to impose some restrictions (for example to prohibit such movement during first 3 years of policy).

Moreover, processes of life insurance company may be more complex than those described by Markov model. For example, let us consider more complex life insurance policy, involving additional Critical Illness benefit and paid up option (see Figure 2).

Figure 2. Modified multiple state model used for calculation of Surplus value



Here again, as in the initial model, while policy stays in "Active" state, premiums P_t are paid to life insurance company at the start of each month. Also when policy moves to state "Surrender" or "Death", surrender value S_t expressed as lump sum (or death benefit D_t) is paid to policyholder at the end of period t. When policy expires at time n insurance benefit, equal to reserve V_n, is paid upon termination.

Additionally, if policyholder suffers from one of diseases described in policy conditions, so-called critical illness (CI) disease, CI benefit is paid and policy moves to state "Critical illness". Despite that it is possible for policyholder to recover from critical illness we will assume that it is not possible to return from here to state "Active" since disease suffered changes significantly transition probabilities to states "Dead", "Surrendered" and "Paid up". While policy stays in "Critical illness" state, premiums P_t still are paid to life insurance company at the start of each month, however premiums may be reduced due to cancellation of CI benefit (benefit was paid, so insurance cover for CI was cancelled). From state "Critical illness" policyholder may move to state "Paid up" without any restrictions usually imposed on such movement.

As it was mentioned above while in state "Paid up" no premiums are paid to insurance company. It may be noticed that behavior of insured who suffered critical illness under some circumstances differ quite significantly from "healthy" policyholders. For example, we observed that about 60% "healthy" insureds who paid up the policy, usually surrender it during following 1-12 months. However this is not the case for insureds who suffered CI before paying up the policy. From this group only about 5% surrender the policy. And it is quite natural since the latter group already used benefits of insurance (and very likely will use it in the nearest future) which is not the case for the first group. So, transition probability from state "Paid up" to state "Surrender" will depend not only on the current state ("paid up") but also on the state preceding current state, so the model looses the above mentioned Markov property at this stage.

If the information about the future of the Markov chain is contained not all in the current state but is in the current and the last state of the process, we call such chain a second order Markov chain and formally we rewrite (2.1) condition:

$$
\begin{aligned}
&P(\xi(t) = j|\xi(t_1) = i_1, \xi(t_2) = i_2, \ldots, \xi(t_k) = i_k) = \\
&\qquad P(\xi(t) = j|\xi(t_k) = i_k, \xi(t_{k-1}) = i_{k-1}) = q_{i_k j}(t_{k-1}, t_k, t).
\end{aligned} \tag{2.4}
$$

The use of higher order Markov chains in insurance problems was discussed also in [1].

If insurance processes (or at least part of them) are modelled using second order Markov chains deterministic calculation of surplus value is not suitable since it is very difficult, if not impossible, to calculate expected profits in (2.2) correctly.

At least partially above mentioned problems of deterministic SV may be addressed through sensitivity testing. However sensitivity testing only allows to estimate deterministic SV under various sets of assumptions but does not provide full distribution of future profits.

In recent years mostly due to the reasons mentioned above more and more attention is paid to stochastic modelling of insurance processes, including calculation of embedded value (see [2], [5], [6], [7], [9]).

For stochastic modelling it is necessary for the company to identify major risk parameters, for example, mortality, investment return, surrenders etc., and possible correlations of them. Simulations of SV are then run over a range of selected risk elements.

For simulations of mortality and morbidity Monte Carlo simulation techniques may be used. Main principles of Monte Carlo simulation methods are outlined in [3], part I, section 5, pages 137 - 147. Generally speaking, series of random numbers are used for each life to determine whether or not death occurred at specific time moment. For a single trial produced random numbers are compared with mortality probabilities and if produced random number is less than "real" mortality probability we assume that death occurred (see also [9]). The same method is used for modelling of morbidity and surrenders. More detailed description of Monte Carlo method as well as other methods used for stochastic modelling of mortality may be found in [2].

Probably the most difficult point is simulation of tail dependencies. Modelling tool called copula may be used in this situation, however it will be very difficult to decide, for example, what kind of copula to use for modelling of dependencies of investment rate and number of surrenders since as it was mentioned above we must take not only economical but also psychological factors into account. We will not deal with modelling of tail dependencies in this paper in more detail.

In the next section we present short description of stochastic modelling of insurance processes in above described case.

3 RESULTS

Here we will consider a group of life insurance policies which involve additional Critical Illness benefit and paid up option as described in Figure 2. We consider that main risk parameters here are mortality, morbidity and surrenders. We use mortality and morbidity tables of Lithuanian population and surrender rates are calculated form company's experience.

We use second order Markov chain (as described above) as a basis for calculations. We use Monte Carlo simulation method for determining number of deaths, surrenders, CI and paid up policies. We did not use stochastic interest rate since investment risk in unit linked insurance is borne by policyholder. One more reason for concentrating attention on mortality, morbidity and surrender risk is that in recent years many papers have been published on the stochastic modelling of interest rates and on returns on assets and on asset - liability modelling (see, for example, [3] part II, section 8, pages 226-263 or [4] where comprehensive overview of Dynamic financial analysis techniques is given). Usually it is assumed that mortality risk may be diversified by pooling of risks, however, we think that there are some situations when adverse experience of mortality may have great impact on company's results, for example, epidemics, terror attacks, limited number of lives exposed to risk, reinsurance etc. The same may be said about morbidity. Therefore, we concentrate on stochastic modelling of mortality, morbidity and surrenders in this paper.

For specimen calculations we used sub-portfolio of 5000 lives, each live was assigned a standard (100%) or extra (greater than 100%) mortality (morbidity) rating. We produced 1000 Monte carlo simulations for each live in the portfolio.

Illustrative simulation results are given in Figure 4. Deterministic Surplus value in this case is 4 447 000 LTL. Produced results may be used for risk management of insurance company, for example, simulation results show distribution of Present value of future profits and p-percentile (for example, $p = 0.05$ of this produced distribution may be regarded as risk measure).

4 CONCLUSIONS and RECOMMENDATIONS

It is obvious that presented model as all other models cannot predict the future. However, simulation results produced using stochastic mortality modelling techniques may provide useful insight into variability of future profits. Presented model shows what might happen if used assumptions will be confirmed in practice, therefore, it is necessary to carefully check all assumptions before model is used in practice. Used assumptions must be periodically checked against experience also.

We dealt only with "natural" variability of mortality (morbidity) and did not take into account possible changes in mortality rates due to, for example, progression in medical aid, improvements in life style and others. Such changes, known as longevity, are especially important for pension annuity business. Introduction of modelling of constant improvements in mortality, named "shock" to the mortality levels, may be found in [2]. Stochastic modelling techniques may be useful for dealing with variabilities in mortality and

/ or morbidity due to the impact of social - economical factors, such as public health policy, underwriting, new treatment methods and others.

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Table 1. Comparison of changes in unit price and annual surrender rates

Period (quarter)	Changes in unit price (USD bonds)	Changes in unit price (World shares)	Surrender rate (USD bonds)	Surrender rate (World shares)
2001 I	4%	-15%	1.00%	2.00%
2001 II	2%	-1%	1.30%	1.70%
2001 III	2%	-14%	1.60%	2.20%
2001 IV	1%	8%	1.99%	2.50%
2002 I	0%	-3%	2.55%	2.30%
2002 II	-11%	-20%	3.10%	2.50%
2002 III	6%	-19%	3.65%	2.70%
2002 IV	-5%	-1%	4.00%	2.90%
2003 I	-3%	-8%	4.60%	2.50%
2003 II	-2%	7%	5.20%	2.44%
2003 III	-4%	3%	5.50%	2.46%
2003 IV	-8%	3%	6.10%	2.20%
2004 I	5%	5%	7.00%	2.50%
2004 II	-2%	1%	5.58%	2.01%
2004 III	1%	-2%	5.05%	1.98%
2004 IV	-9%	1%	4.77%	2.01%

Figure 3. Analysis of changes in unit prices (in percentages)

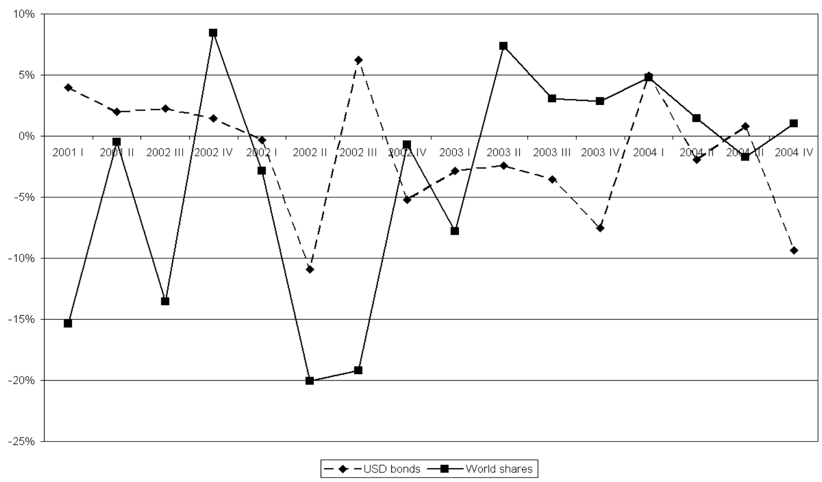

Figure 4. Distribution of Surplus value - results of stochastic simulation