APPROXIMATION MODEL FOR THE ADDITIONAL PENSION CAPITAL DYNAMICS

[1]Gribkova Valentina, [2]Matvejevs Andrejs and [3]Pavlenko Oksana

[1] Dr.sc.ing., Insurance company "BALVA"
36, Kr.Valdemara str., Riga LV-1050,
Tel: +3717506957,
Fax: +3717506956,
E-mail: director@balva.lv

[2] Dr.sc.ing., Probability Theory and Mathematical Statistics Dept., Riga Technical University, Meza iela ¼, Riga, LV-1048,
Tel: +3717089558,
Fax: +3717089580,
E-mail: matv@tvnet.lv

[3] Dr.math., Probability Theory and Mathematical Statistics Dept.,
Riga Technical University,
Meza iela ¼, Riga, LV-1048,
Tel: +3717089558,
Fax: +3717089580,
E-mail: oks@tvnet.lv

Abstract

The purpose of the article is to construct models to analyse and forecast the accumulated additional pension capital. This is necessary for estimation of parameters of pension fund's stability. The *Financial and Capital Market Commission* has proposed the formula for calculation of the pension capital value for each quarter of the year. The index of R as the operation result of the pension plan was included in this formula as known value. But in a real time model this coefficient can vary by the different ways according to the pension plan conditions. So the main aims of the article are following.

1. To construct the model for R as time series, to analyse it's dynamics and to construct the appropriate forecast.
2. To choose the asymptotic distribution for R and then to make interpolation of the pension capital.

This analysis allows *Financial and Capital Market Commission* as of supervising organization to have more possibilities for control of the pension fund.

Keywords: Pension Fund, Additional Pension Capital, ARMA Model, Dynamical Forecast, Local Regression

1. INTRODUCTION

There is very quick development of Private Pension Fund (PPF) business in Latvia in last few years. Only in year 2001 PPF contributions have rose 2.9 times and received 4,807 millions Lats (about USD 8 millions). In PPF growing also heighten necessity of qualitative computer programs for accounting purposes. More ever computer programs must include not only these technologies that are used in Latvia by today. Programs must include technologies that are unfamiliar for Latvia now but will be meaning at next future. Thereby this approach can stimulate rapid growth of pension business as it was in Great Britain in 1950ties. In those days parallel with developing of computer sciences Pension funds offered to theirs clients new derivative pension fund products, which stimulate quick growing of PPF assets.

To establish software according these requirements is essential to look through algorithms applied in other countries and especially in UK. Then these algorithms can be adapted to Latvia market according Latvia market's requirements and local legislation. Valuation of PPF additional capital is one of the essential parts of PPF valuation and we will concern to it. The aim of this article is to adapt these algorithms to real time model and to estimate acquired model.

2. STATEMENT OF THE PROBLEM

To calculate the saved additional pension capital in Pension Fund, payments of the participants pension plan recalculate as determined numbers of a share (unit), which are equated to the relation between value of payments and meaning of one share appropriate pension plan by the beginning of a quarter. Hence, it is possible easily to calculate value of a money sum, which collects on individual account of each participant for certain time.

The additional pension capital accumulated in the individual account of a pension plan participant shall be calculated at the end of each quarter, based on the profit-and-loss account of the pension plan during the particular quarter:

$$k_b = k_s + c - e + \frac{k_s + c - e}{K + A} \cdot R \qquad (1)$$

where

k_b - the additional pension capital accumulated in the individual account of the pension plan participant at the end of the current quarter;

k_s - the additional pension capital accumulated in the individual account of the pension plan participant at the end of the preceding quarter;

c - instalments made by the pension plan participant and instalments made for his/her benefit in accordance with the pension plan, as well as wire transfers from another pension fund or pension plan in the respective quarter;

e - administrative costs applicable to the pension plan participant in the respective quarter;

K - item "Pension plan capital in the accounting period" of the profit-and-loss account of the pension plan in the respective quarter;

A - item "Instalments and disbursements of the pension plan" of the profit-and-loss account of the pension plan in the respective quarter;

R - item "Pension plan operation result" of the profit and loss account of the pension plan in the respective quarter.

In focus of technology is dividing of investment to equal parts, which are called for units. After this the goal is to seek count of units corresponding to each unit holder. As new member enters to PPF to him sold part of units by bid price and after this his unit portfolio will rise respectively by rise of all PPF investment portfolio. When this member dissolves from the plan, then units will be sold to him by current offer price. After this we are going to search and estimate regression equation of this model.

3. THE DEFINITION OF THE INDIVIDUAL PRICE

The main approach in consideration of this problem is the distribution of the investments in equal shares (or uniform price) on all participants of pension fund. The purpose of such approach - to define that part of means of fund, which belongs to each participant, and also to supervise process of change of this share at reception of the new participants or leaving of the participants.

A major task of definition of the additional pension capital is of observance of interests of the participants (fair distribution of results of the investment income). Inadmissible the situation is, when the participants who have entered fund in the beginning of its activity, are compelled to pay the initial charges of fund. The ideal decision of this problem would be distribution of these charges of fund to all participants fifty-fifty.

4. METODOLOGY

According with initial task we should apply the formula (1) for recalculation "on-line" mode, but not only in the respective quarter. We can rewrite k_b as k_t and k_s as k_{t-1}, where k_{t-1} means the pension capital calculated for the prior time and $k_0=0$.

Due to $K = \sum_i k_s$, where i means PF participant with number i, then we can rewrite

$$K = \sum_i k_{t\,i.}$$

We know, what administrative payments are connected to each participant of the pension plan during one period (for example, within one year). It is so-called periodic payments. Further, from the formula (1) we know also what with each participant so-called expendable payments (for example, payments connected to expendable transfer of money to the account of pension fund) are connected. Therefore at any time moment t the current administrative and other payments of one client can be define, carrying out linear interpolation to periodic payments, and also summarizing expendable payments, therefore we can rewrite e as e_t.

As similar a total sum of payments in fund and the general payments from fund can be defined if to summarize payments and payments for each participant of pension fund separately, i.e. A means A_t.

Thus, we still had the unique factor R, which is necessary for defining in a mode on-line. It is difficult enough to define this variable precisely at any moment of time; therefore here it is necessary to apply the approached methods. In effect of calculation R_t is connected to other very important question of work of pension fund. Namely, a bigger problem relates to the fact that the calculations consider only revenue flows in the funds (contributions, investment income, expenses). There is no reflection of capital gains (realised or unrealised).

Such investments of the capital are complex for estimating in a mode on-line.

In a basis of calculation of additional pension capital accumulated in the private pension fund on the individual accounts a principle of recalculation of money sums through units is incorporated. Before now this process is applied to account of residuals on the end of a quarter and on the end of a year. The main idea of a procedure of recalculation consists in following:

1. By the beginning of the first time period cost one unit is equal 1 Lats.
2. Unit prices are calculated quarterly.
3. Units are allocated at the next price calculated after receipt of contribution or due date of payment of claim.
4. Initial unit price is 1 Lats, which is used to allocate units for the first three months (of a year) contributions paid. Subsequent contributions are allocated at the unit price next following receipt of premiums.

Now the aim is to adapt these algorithms to real time model and to estimate acquired model.

So, having entered the earlier stipulated designations for the formula for calculation of the additional pension capital accepts the following kind.

$$K_t = K_{t-1} + c_t - e_t + \left(\frac{K_{t-1} + c_t + e_t}{\sum K_{t-1} + \sum (c_t - e_t)} * R_t \right) \qquad (2)$$

For the profit and loss factor $\boldsymbol{R_t}$ it can be rewritten as:

$$R_t = (K_{t-1} + c_t - e_t) \left(\frac{K_t}{\frac{n_t}{n_{t-1}} K_{t-1} + c_t - E_{adm}} - 1 \right) \qquad (3)$$

Here $\boldsymbol{c_t}$ - contributions of the participant**;**

$\boldsymbol{e_t}$ - payments by the participant**;**

E_{adm_t} - administrative charges.

5. DISCUSSION AND RESULTS

To our attention the following data on the participants of pension fund in the period with 01.01.2001 up to 31.03.2004. (see table 1) are offered.

Table 1. Data for the period from 31.12.2000 to 31.03.2004

t	**n_t**	**K_t**	**c_t**	**e_t**	**E_adm_t**	**R**
Date	**Count of individuals**	**Pension plan capital**	**Contribution of the partic.**	**Payments by the particip.**	**Administr. charges**	
31.12.2000	6992	5874134	1667553	39980	45013	
31.01.2001	7040	5978075	103941	720	346	720
28.02	7238	6175568	197493	1416	1205	1416
31.03	7247	6409806	212393	27736	13016	49480
30.04	7287	6534326	124520	12546	849	12544
31.05	7337	6671253	136927	20348	1541	20343
30.06	7430	6877912	137435	25730	12825	94774
31.07	7625	7025788	147876	9345	4812	9339

Table 1. Data for the period from 31.12.2000 to 31.03.2004 - Continued

31.08	8861	7275281	249493	11724	2198	11720
30.09	9121	7423404	156746	6819	11336	-1801
31.10	9204	9009958	1586554	19348	5376	19336
31.11	9458	9248863	238905	33982	7895	33953
31.12	17359	9496371	83326	82459	11444	246336
31.01.2002	17409	9790617	294246	22286	11349	22260
28.02	17683	10125980	335363	2125	4360	2124
31.03	17768	10666049	232958	8597	6466	315511
30.04	17844	10783216	305105	17244	8224	-170566
31.05	18105	10993145	318078	19085	7806	-89001
30.06	18595	11723084	307285	14726	8427	437053
31.07	18880	11961856	290334	20228	9305	-31310
31.08	18940	12045323	310105	31044	8796	-195453
30.09	19233	12736861	463932	13507	9482	240930
31.10	20041	12916543	335210	18003	9101	-137429
31.11	20050	13246033	370040	9361	8836	-31168
31.12	20064	13972331	303502	3792	9890	426277
31.01.2003	21101	14126135	356029	20018	9556	-182085
28.02	21836	15001963	302885	15341	9004	587917
31.03	22198	15163201	409978	14654	9575	-233940
30.04	23011	15948316	398875	25388	10035	411362
31.05	23880	16244450	401022	36844	9963	-68002
30.06	24201	16639282	405117	42757	11227	32450
31.07	24800	17162433	420010	43200	11244	146244
31.08	25014	17544201	445366	40115	10618	-23469
30.09	25244	18012652	394680	38485	11837	112182
31.10	25936	18399062	400350	31878	11887	17926
31.11	26218	18876502	397240	36224	12122	116349
31.12	26610	19315385	406473	46624	11935	78985
31.01.2004	27300	20067037	445560	25118	13244	330988
28.02	28775	20968846	480240	31876	13118	453155
31.03	30084	21419863	389589	49028	12422	110392

The respective graph of profit/loss statement is shown in Figure 1.

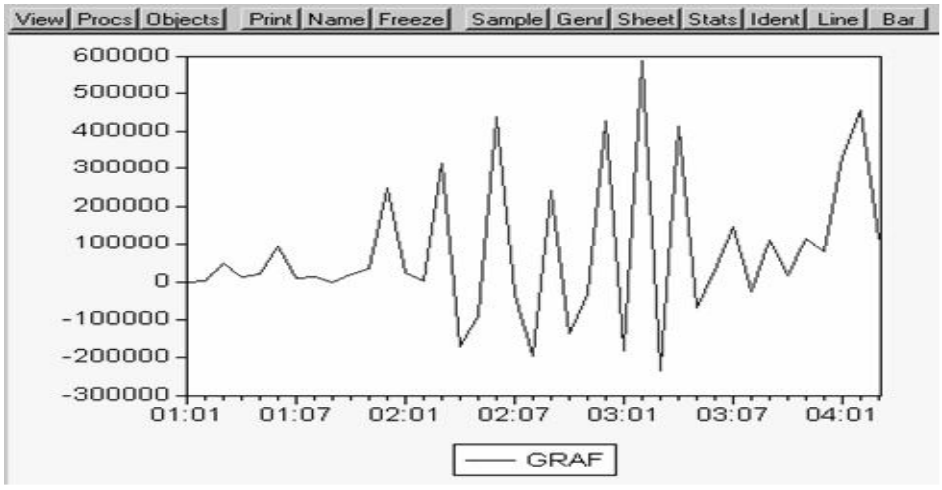
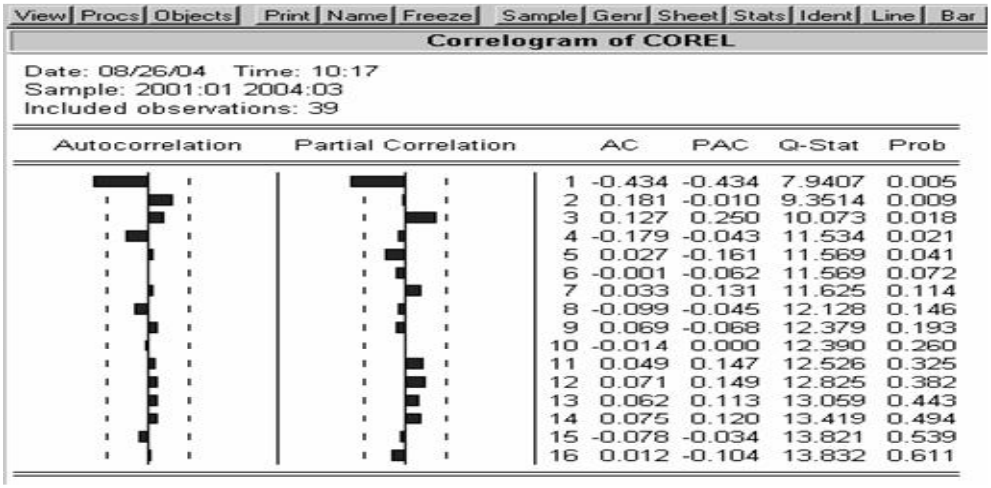


Figure 1. The graph of the pension fund's operation result.

For further analysis we have used E-VIEWS software.
To find the best ARMA model we have established corellogram to our time series. From Figure 2 we can easily see that possible models are AR(1), MA(1) or ARMA (1) models.



	AC	PAC	Q-Stat	Prob
1	-0.434	-0.434	7.9407	0.005
2	0.181	-0.010	9.3514	0.009
3	0.127	0.250	10.073	0.018
4	-0.179	-0.043	11.534	0.021
5	0.027	-0.161	11.569	0.041
6	-0.001	-0.062	11.569	0.072
7	0.033	0.131	11.625	0.114
8	-0.099	-0.045	12.128	0.146
9	0.069	-0.068	12.379	0.193
10	-0.014	0.000	12.390	0.260
11	0.049	0.147	12.526	0.325
12	0.071	0.149	12.825	0.382
13	0.062	0.113	13.059	0.443
14	0.075	0.120	13.419	0.494
15	-0.078	-0.034	13.821	0.539
16	0.012	-0.104	13.832	0.611

Figure 2. The corellogram of the time series R

After estimation and comparing these models, we found that the best model is AR(1).
After that we have analysed season effects on our model. We have find that the model has small seasonally changes in winter times. This finding led us to introduce new formula (4):

$$R_t = C + \alpha_1 r_{t-1} + \alpha_2 S_t + \varepsilon_t \,, \qquad (4)$$

$$S_t = \begin{cases} 1, & w\text{int}er\ months \\ 0, & non\quad w\text{int}er\ months \end{cases}$$

Here C, α_1 and α_2 are coefficients of the AR(1) model, ε_t are residuals.
We have estimated model's stationary with *Unit root test* and found that the time series for R has not unit root.

Also we have tested model's stability with Chow test – Figure 3.

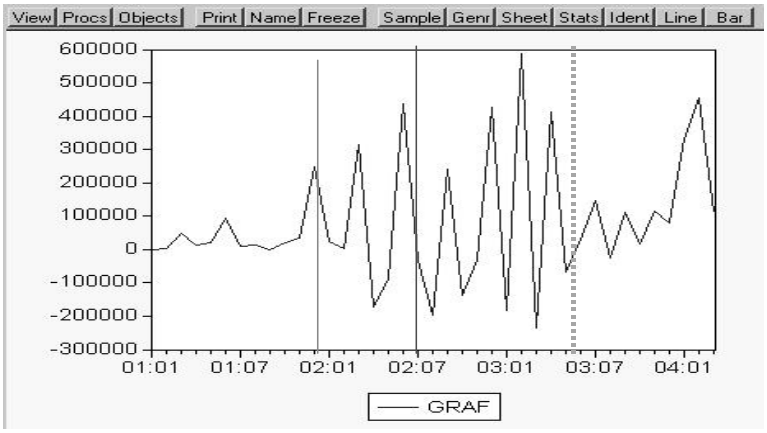


Figure 3. The graph of the Chow test's procedure.

Firstly we have divided model in two equal parts and did not find any changes. After that we have analysed changes in two most critical moments January 2001 and July 2003. We

have found that model have changes in July 2003 (dashed line in Chart 3) that it is due beginning of trend at that moment.
Financially we can explain these trends with funds developments. In first time (year 2001) in market there were only few participants, which led to high fluctuations in funds profit. Also there are large influences in middle period (year 2002-2003) because new funds come into market and always in beginning of operations fund results were unstable. But in the last period from July 2003 we can find stabilisation of market and also profit trend with tendency to grow up.
In the Figure 4 we can see graph for functions with trend and without trend. We easily can see, that the main difference between functions is in last period.

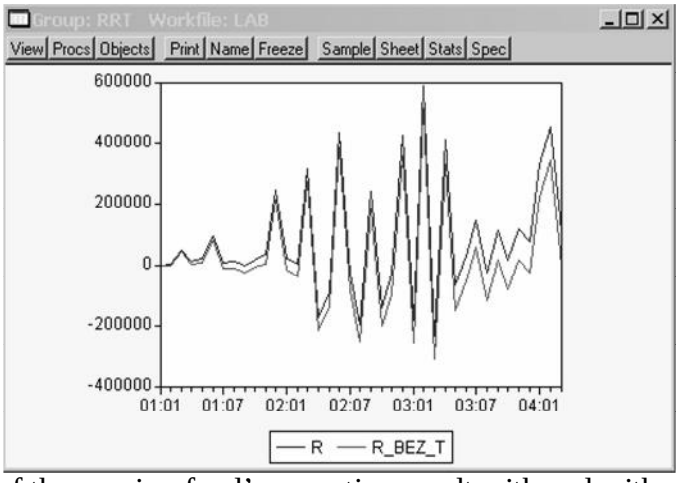


Figure 4. The graph of the pension fund's operation result with and without trend.

We have found trend in last period also with method of local regression – Figure 5

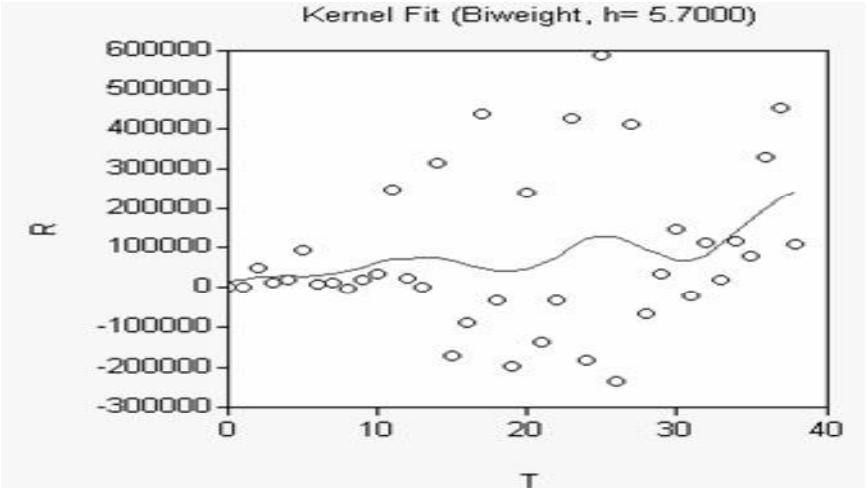


Figure 5. The graph of the local regression analysis for the R.

After that we have forecasted future R values in Figure 6. We can see that forecast in slow but profit growing.

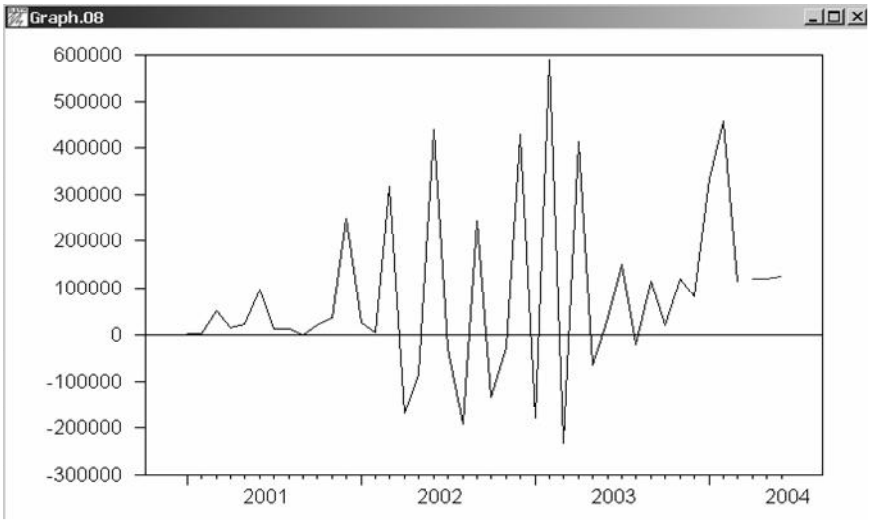


Figure 6. The graph with the time series R forecasting

6. CONCLUSION

We have found and estimated autoregressive model for pension fund additional capital in real time condition which have following characteristics:

- time series in last periods have trend;
- time series have seasonality;
- time series are stationary;
- time series did not have heteroskedasticity;
- model is adaptive.

The algorithm that we have used is good tool for pension fund stability analysis. Supervisory authorities can use it as procedure for particular pension fund stability analysis or overall pension fund market tendencies analysis.

REFERENCES

1. Enders W. RATS Handbook for Econometric Time Series. John Wiley & Sons Inc. New York. 1996..
2. Matvejevs A. Private Pension fund in Latvia, Vilnius University. Diploma paper. 2001.
3. Winklevoss H. E., Pension Mathematics with Numerical Illustrations. Richard D. Irwin, Inc., Homewood, Illinois, 1977.