

MANAGEMENT STRATEGY FOR SYSTEMICS RISK WITHIN INSURANCE

CONTRACTS:

THE CASE OF FARM'S CROP INSURANCE

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Abstract

Because of the strong systemic component of farm risks, the annual Loss Ratio of the insurer will be extremely variable around the balance. Then, the systemic risk component is a major obstacle that limits the agricultural insurance emergence. The aim of paper is to present an approach to overcome such obstacle.

This paper approach is mixing the pooling technique usually used to manage insurance contracts and the transfer of the insurer risk to futures markets as used to manage systemic risks. It is applied to a crop yield insurance but can be extended to other insurance contracts such as revenue or crop margin. This approach can also be widen to all insurance policies which included a large systemic risk component correlated with one or more futures (energy futures, weather derivatives...).

This original form of securitization was tested using 1972-2002 corn yield data of US State of Illinois and the 1996 & 1997 Chicago Board Of Trade corn yield and price futures data.

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1 INTRODUCTION

Our paper aims at dealing with the ability for an insurer to design and manage a systemic risk within insurance contract. It presents an original form of securitization that transfers insurer's risks to futures market. It is applied to a crop yield insurance but can be extended to other agricultural insurance contracts such as revenue or crop margin. This approach can also be widened to all insurance policies which included a large systemic risk component correlated with one or more futures (energy futures, weather derivatives. . .).

In recent research in agricultural economics, crop yield insurance and revenue insurance are a major topic of discussion. Four main research areas are mentioned. First of all, the farm risk management using insurance contracts is dealt with in literature, see for example Gloy and Baker (2001, 2002) as well as Coble et al. for modelisation or Sherrick et al. for an empirical analysis of farmer's demand. Secondly, it proposes several models to estimate the contracts premiums (Stokes; Just et al.). Thirdly, it proposes a definition of optimal insurance contracts (Mahul and Wright). Finally, some authors such as Skees and Barnett deal with the role and implication of government. But today we can observe that crop yield insurances in the United States of America or in the European Union do not exist without government reinsurance and/or subsidies.

Agricultural risks are multidimensional. They include price, yield, quality and production cost hazards. Moreover, they include a high systemic risk component because of the high correlation among farm-level risks. Consequently, individual risks are not independent and the law of large numbers does not apply. Whatever the contract design or the premium, the annual Loss Ratio of insurer will be extremely variable around the balance. Therefore, if an insurer pools a portfolio of several crop insurance contracts, it will bear the systemic risk component. It does not usually have enough equity to face up to this risk (Smith et al.).

Miranda and Glauber (1997) argue that the systemic risk component is the major obstacle that prevents an independent private crop insurance industry from emerging. We agree with this statement, and this paper aims at overcoming the difficulty.

The first idea is to have multi-year management to get the balance of the Loss Ratio over several years. The law of large numbers over several years applies in this reasoning. It implicitly assumes that the risk is independent from one year to another and that there is neither a change in the climate nor an evolution of the technical and economic environment. In this context, some authors determine a contract that maximizes the profit utility of the insurer (Nelson and Loehman; Knight and Coble; Ker and Goodwin). Others even argue such as Skees and Barnett that crop yields are not insurable, which results in the need for public action.

Private reinsurance is the second idea to cope with this difficulty. Even at this level, the pooling of production regions cannot be really achieved because the regions are heterogeneous. The weather and technical and economic hazards can be very different from one region to another irrespective of the size of the region (Turvey et al.; Mason et al.).

A third idea is the transfer of the systemic component to financial markets, as suggested by Miranda and Glauber: *“Clearly, neither insurance markets nor options markets alone are capable of providing adequate individual crop loss risk protection in the absence of government support. However, insurance and option [or futures] markets together, each performing within its inherent limitations and exercising its own particular strengths, could provide a market solution to the crop risk insurability problem.”*

Considering the literature review of Tomek and Peterson and Collins, the scientific community deals abundantly with the use of futures and options by optimal hedging. For example, to manage farm revenue risk, Vukina et al. propose double hedging with price and yield futures.¹ Even in ideal conditions, double hedging is not able to eliminate the risk generated by the covariance between price and yield. Opposed to this discrete time strategy, Guinvarc’h et al. propose continuous management strategy able to eliminate this “covariance” risk component. It is based on the replicating portfolio of the revenue futures contract and uses both the crop yield futures contract and the price futures contract. This strategy is applied by an intermediary that offers a revenue futures or option contract.

Mason et al. propose to manage insurer (or reinsurer) risk with optimal dual hedging. They test this proposition in the case of Risk Management Agency’s reinsurance. It results that RMA’s risk is indisputably reduced but the risk taken remains too high for a private insurer.

We choose to pursue this channel of research into the risk management problem of the crop yield insurer. While others have argued that it is impossible for a private insurer to offer insurance contracts that can deal with the multidimensional risks farmers face without government support, we theoretically show that an insurer can manage a crop yield contract that includes both systemic and idiosyncratic risk components. The first sections present the theoretical approach of the model and the second realizes the tests.

2 THE THEORETICAL APPROACH

Our approach to crop yield insurer management is described in figure 1. The insurer sells an insurance contract to the farmer, conserves the idiosyncratic risk component and uses financial markets to transfer the multidimensional systemic risk component.

The first subsection defines the model and its assumptions. The second subsection proposes an estimation of the crop yield insurance premium. Using this estimation, a replicating portfolio is built in the third subsection allowing a self-financed strategy to manage the insurance contract.

2.1 The model and its assumptions

In order to develop the crop yield insurance contract, we first describe the farmer’s financial loss brought about by the crop yield and then the contract indemnity. Next, the model of financial

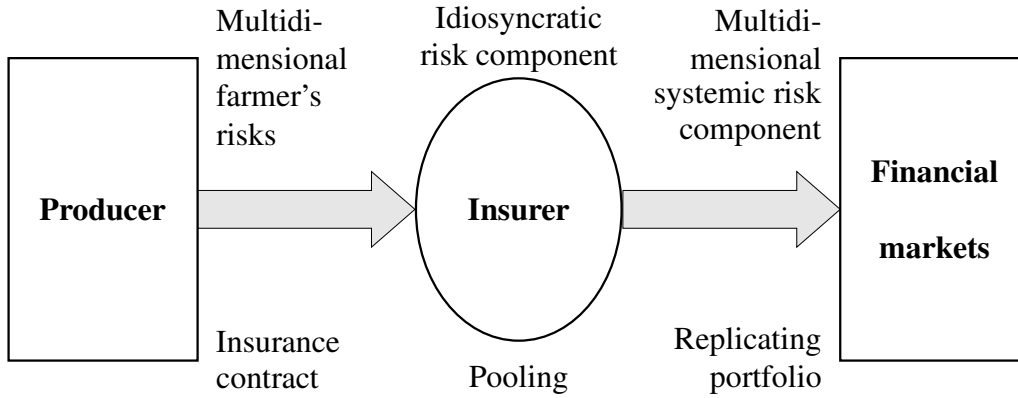


Figure 1: Illustration of the insurance contract management

markets is presented. It includes three contracts: the price futures, the crop yield futures and the zero-coupon bond. We define the time decomposition of the risk management of the crop yield insurance contract. At last, we define the assumptions on the conditional density of losses.

2.1.1 The insurance contract

In the case of a crop yield insurance contract, the farmer's indemnity depends on the potential financial loss suffered because of a low crop yield. We assume that the insurer insures n farms for this contract. The farmer j has a crop yield loss if his crop yield y_j is smaller than a minimum y_m^j . The quantity y_m^j is the smaller crop yield suitable for the farmer. The financial value of this loss ℓ_j is equal to $\ell_j = p_j \times \max(y_m^j - y_j, 0)$ where p_j is the farm random unit price of the product at the end of the production period (T^+). Therefore, the concept of *crop yield loss* includes both price hazard and crop yield hazard.

The mathematical indemnity function $I(\ell_j)$ defines the insurance contract. The indemnity principle² requires that $0 \leq I(\ell_j) \leq \ell_j$. Moreover, the model is designed for an indemnity proportional to the farm's loss. Then, $I(\ell) = \lambda \ell$ where λ is a constant value in $[0, 1]$. In the model, the policy holders are uniformly distributed throughout the areas linked to the crop yield futures. It is assumed there is no moral hazard.

Let us also specify two differences between a crop yield option and a crop yield insurance contract: first of all, a crop yield option is unidimensional when crop yield insurance is bidimensional (it does not depend on the random price). Secondly, a crop yield option does not depend on farm crop yield but on aggregate crop yield. The current Multiple Peril Crop Insurance (MPCI) indemnities are also different from the crop yield insurance contract definition because they do not include the random dimension of the price.

2.1.2 The financial contracts motions assumptions

We note B_t a zero-coupon bond with the continuous risk-free rate r . We note t the time between 0 (the beginning of the quotation of both futures) and T (the settlement time). W_t^F and W_t^Y stand for two one-dimensional standard Brownian motions defined on filtered probability space

$(\Omega, \mathcal{F}, \mathbb{P})$. F is the price futures contract and Y is the crop yield futures contract. At maturity, the price of the crop yield contract is proportional to the area crop yield. We assume that F and Y are geometric Brownian motions.³ The parameters of F are μ_F and σ_F and the parameters of Y are μ_Y and σ_Y . Let us present the model of the financial market:

$$F_t = F_0 + \int_0^t \sigma_F F_u dW_u^F + \int_0^t \mu_F F_u du \quad (1)$$

$$Y_t = Y_0 + \int_0^t \sigma_Y Y_u dW_u^Y + \int_0^t \mu_Y Y_u du \quad (2)$$

$$B_t = \exp(-r(T-t)) \quad (3)$$

where $\mu_F, \mu_Y, \sigma_F > 0$ and $\sigma_Y > 0$ are known constants. The price motion of F and Y are not independent so we note $\delta = \text{cov}(W_u^F, W_u^Y)$. We assume that the covariance δ between both Brownian motions is negative because generally the price increases when production decreases.

We define \mathbf{W}_t a two-dimensional Brownian motion by:

$$\mathbf{W}_t = \begin{pmatrix} W_t^F \\ \frac{1}{\sqrt{1-\delta^2}}(W_t^Y - \delta W_t^F) \end{pmatrix}$$

By applying Girsanov's theorem to W_t with the risk neutral probability,⁴ we get W_t^* (Musiela and Rutkowski, 1997, section 10.2):

$$\mathbf{W}_t^* = \mathbf{W}_t - \int_0^t \begin{pmatrix} \sigma_F & 0 \\ \frac{-\delta\sigma_F}{\sqrt{1-\delta^2}} & \frac{\sigma_Y}{\sqrt{1-\delta^2}} \end{pmatrix} \begin{pmatrix} r - \mu_F \\ r - \mu_Y \end{pmatrix} du \quad (4)$$

It results that, under \mathbb{P}^* :

$$d \begin{pmatrix} F_t \\ Y_t \end{pmatrix} = \begin{pmatrix} \sigma_F F_t & 0 \\ 0 & \sigma_Y Y_t \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \delta & \sqrt{1-\delta^2} \end{pmatrix} .dW^* + r \begin{pmatrix} F_t \\ Y_t \end{pmatrix} dt \quad (5)$$

2.1.3 The time decomposition

The risk management from 0 to T can be split into two periods. We introduce the time $T + \varepsilon$, that stays after of T for a very little period ε . The first period begins at 0 and ends at T and the second period begins at T and ends at $T + \varepsilon$.

We note T^+ the ‘‘right-limit’’ of $T + \varepsilon$ when $\varepsilon \rightarrow 0$. At time T , we assume that the individual results (p_j, y_j) are not known. They are only known at T^+ and as a result, the individual loss ℓ_j can be calculated.

In terms of model building, we use a retrospective reasoning (from T^+ to T and from T to 0). In terms of insurer strategy management, it is naturally the reverse. Then, in the second management step, we build an instantaneous insurance contract that begins at T and finishes at T^+ . Its premium $Pr^j(F_T, Y_T)$ is the conditional expected value at T of the indemnity j

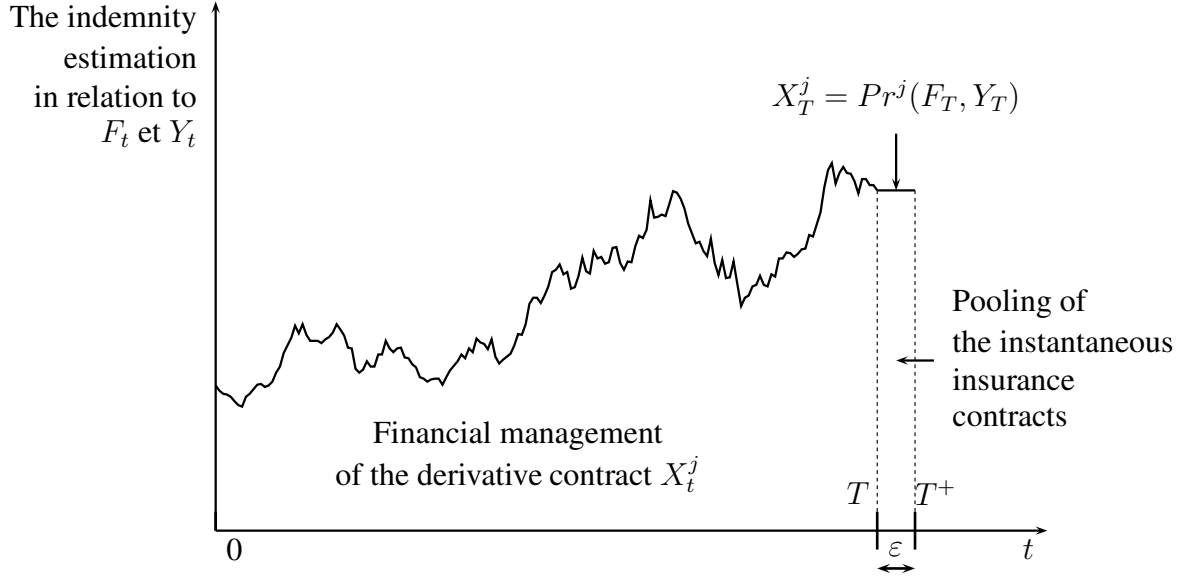


Figure 2: The illustration of the crop yield insurance contract management

knowing (F_T, Y_T) . Then, the premium of the instantaneous insurance contract depends on F_T and Y_T . It is a common insurance contract managed in a very short time (ε) . Next, in the first management step, we build a financial contract X^j bought at $t = 0$ whose price at T accurately tallies with the premium of the instantaneous insurance contract $Pr^j(F_T, Y_T)$. It is a derivative contract defined by its underlying assets (the crop yield futures, the price futures) and its price at maturity ($Pr^j(F_T, Y_T)$ at T^-).

At $t = 0$, the farmer is subscribing *an insurance contract* that, if needed, gives an indemnity at T^+ that depends on this loss. It is the view of the insured farmer. The insurer sells the farmer *the financial contract* X_j (at $t = 0$) whose value allows the financing of the *instantaneous insurance contract* premium at T (figure 2) that, if necessary, provide an indemnity with the farmer at T^+ .

2.1.4 The conditional density of losses

The moment T is the hinge between the two management operations. At this time, the insurer needs an estimation of the instantaneous insurance contract premium. Therefore, we introduce the conditional density f_{F_T, Y_T}^j of ℓ_j which depends on F_T and Y_T . We assume that f_{F_T, Y_T}^j is known and is twice differentiable in the two variables.

We deduce the conditional density of loss from two assumptions. The first one concerns the relation between y_j and Y_T . Ramaswami and Roe analyze different models that has been used to represent this relation. They support the dominance of the multiplicative form over the additive one. However, following numerous authors (Just et al.; Mahul and Wright; Smith et al.), we choose the most classical linear additive form of modelisation:

$$y_j = \alpha_j + \beta_j Y_T + \gamma_j \zeta \quad (6)$$

where α_j , β_j and γ_j are parameters of farm j and where ζ is a random variable with $\mathbb{E}[\zeta] = 0$. By model building, the random values ζ and Y_T are independent. Moreover, we assume that ζ is a standard normal random variable.

On the one hand, the crop yield is always positive and on the other hand loss is defined only if crop yield is less than y_m^j . Therefore, the loss exists if $0 < y_j < y_m^j$. We deduce that loss exist if ζ verifies:

$$\kappa^j = \frac{-\alpha_j - \beta_j Y_T}{\gamma_j} < \zeta < \frac{y_m^j - \alpha_j - \beta_j Y_T}{\gamma_j} = \eta^j$$

The second assumption concerns the farm price at time T^+ . We assume that the farm crop price p_j is the future price F_T . It results that $\ell_j = F_T \times \max(y_m^j - y_j)$. Therefore, the indemnity does not take account of the farm price basis risk in this model.

From Equation 6 and the ℓ definition, we conclude that the conditional density of losses f_{F_T, Y_T}^j is a normal density with mean $F_T(y_m^j - \alpha_j - \beta_j Y_T)$ and standard deviation $F_T \gamma_j$.

2.2 Estimation of the insurance contract premium

First, we calculate the premium $Pr^j(F_T, Y_T)$ of the instantaneous insurance contract at time T . This conditional value allows us to define the settlement price of the derivative. Second, we calculate the derivative price at 0 and deduce the crop yield insurance contract premium.

2.2.1 The price of the instantaneous insurance contract

The price of the instantaneous insurance contract is defined by:

$$Pr^j(F_T, Y_T) = \mathbb{E}_T[I(\ell_j)|F_T, Y_T]$$

Using the conditional density function, the expected indemnity may be calculated as follows:

$$\begin{aligned} Pr^j(F_T, Y_T) &= \int_0^{+\infty} f_{F_T, Y_T}^j(\ell_j) I(\ell_j) d\ell_j \\ &= \int_{\kappa^j}^{\eta^j} F \lambda (y_m^j - \alpha_j - \beta_j Y - \gamma_j \zeta) f(\zeta) d\zeta \\ &= F \lambda (y_m^j - \alpha_j - \beta_j Y) \int_{\kappa^j}^{\eta^j} f(\zeta) d\zeta \\ &\quad + F \lambda \gamma_j \int_{\kappa^j}^{\eta^j} -\zeta f(\zeta) d\zeta \end{aligned}$$

where f is the standard normal density function. We observe that $f'(\zeta) = -\zeta f(\zeta)$ and obtain the estimated premium of the instantaneous insurance contract:

$$Pr^j(F_T, Y_T) = F \lambda (y_m^j - \alpha_j - \beta_j Y) (N(\eta^j) - N(\kappa^j)) + F \lambda \gamma_j (f(\eta^j) - f(\kappa^j)) \quad (7)$$

where N is the normal standard cumulative function.

2.2.2 The price of the derivative contract

The price of the derivative contract X^j is equal to $Pr^j(F_T, Y_T)$ at T . We know that $X_T^* = e^{-rT} Pr^j(F_T, Y_T)$ is a square-integrable random variable under the martingale measure \mathbb{P}^* . Therefore, from the martingale representation property, we conclude that there exists one and only one predictable process θ such that the stochastic integral

$$X_t^* = \mathbb{E}_{\mathbb{P}^*}[X^*] + \int_0^t \theta_u dW_u^* \quad (8)$$

follows a (square-integrable) continuous martingale under \mathbb{P}^* . We deduce that the value X_t of the derivative at t is equal to $e^{rt} \mathbb{E}_{\mathbb{P}^*}[X^* | \mathcal{F}_t]$. This value must be estimated using numerical procedures because an explicit formula is not available.

The premium of the crop yield insurance contract at subscription time t_0 is equal to the value $\pi_{t_0}(X)$ of the derivative. Moreover, we note that the premium depends on the price of futures contracts. The premium can differ from one year to another. For earlier subscriptions, markets have less information, so anticipations are just based on historical grounds. Therefore, if the subscription is early (at time 0), the premium should be stable from year to year.

Now, we know the price of the crop yield insurance contract and the next subsection presents the insurance contract management strategy.

2.3 The insurance contract management strategy

As presented in figure 2, the insurer breaks down the crop yield insurance contract management into two steps. This subsection presents both the pooling step and the financial step.

2.3.1 Pooling of the instantaneous insurance contract step: from T to T^+

At T , the insurer receives the premium $Pr^j(F_T, Y_T)$ to pool farmers' risks at T^+ . F_T and Y_T are known. Then, for the n insured farms, we get:

$$E \left[a_j \sum_{j=1}^n I(\ell_j) \middle| F_T, Y_T \right] = \sum_{j=1}^n a_j E[I(\ell_j) | F_T, Y_T] = \sum_{j=1}^n a_j Pr^j(F_T, Y_T)$$

where $(a_j)_{0 \leq j \leq n}$ represent the respective insured surface of the n farms. If assumptions are verified, the error pooling is:

$$\sum_{j=1}^n a_j \gamma_j \zeta_j$$

For all Y_T , we note that $\sum_{j=1}^n y_j$ converges to Y_T in probability when n increasing because the n policy holders are uniformly distributed throughout the area. Because of the Independence

between Y_T and $(\zeta_j)_{j=1,\dots,n}$, it results that error pooling⁵ converges in probability to 0.

2.3.2 Financial step: from 0 to T

The quotation of X^j on the market is not feasible first because this derivative contract is more complex than an European option. Second, the derivative is specific to the insurer, limiting its potential liquidity. Thus, the insurer has to manage the derivative directly on the two futures F_t and Y_t (delta hedging). Then, in order to describe the management strategy from 0 to T , its replicating portfolio should be built.

A portfolio is described by the line matrix (ϕ_F, ϕ_Y, ϕ_B) where ϕ_F is the number of price futures contracts, ϕ_Y is the number of crop yield futures contracts and ϕ_B is the number of unit bonds.

Proposition 1 *The self-financing replicating portfolio ϕ of X^j is given by:*

$$\phi_u = \left(\frac{\partial X_u}{\partial F}, \frac{\partial X_u}{\partial Y}, X_u - \frac{\partial X_u}{\partial F} F_u - \frac{\partial X_u}{\partial Y} Y_u \right) \quad (9)$$

Proof.

On the one hand, Equation 8 gives:

$$dX_u^* = \theta_u dW_u^* = h_u \cdot \begin{pmatrix} dF^* \\ dY^* \end{pmatrix} = h_u \cdot \begin{pmatrix} dF \\ dY \end{pmatrix} - r h_u \cdot \begin{pmatrix} F_u \\ Y_u \end{pmatrix} du \quad (10)$$

where we have put

$$h_u = \exp -r(T - u) \theta_u \begin{pmatrix} 1 & 0 \\ \frac{-\delta}{\sqrt{1-\delta^2}} & \frac{1}{\sqrt{1-\delta^2}} \end{pmatrix} \begin{pmatrix} 1/\sigma_F F_u^* & 0 \\ 0 & 1/\sigma_Y Y_u^* \end{pmatrix}$$

Knowing that $dX_u^* = dX_u - rX dt$, it results that $\phi_B = X_u - h_u \cdot \begin{pmatrix} F_u \\ Y_u \end{pmatrix}$. On the other hand, we applied Itô's formula to X :

$$\begin{aligned} dX_u &= \frac{\partial X_u}{\partial u} du + \frac{\partial X_u}{\partial F} dF_u + \frac{\partial X_u}{\partial Y} dY_u + \frac{1}{2} \frac{\partial X_u}{\partial F \partial F} (\sigma_F F_u^*)^2 du \\ &+ \frac{\partial X_u}{\partial F \partial Y} \langle (\sigma_F F_u^*, 0), (\delta \sigma_Y Y_u, \sqrt{1-\delta^2} \sigma_Y Y_u) \rangle du + \frac{1}{2} \frac{\partial X_u}{\partial Y \partial Y} (\sigma_Y Y_u^*)^2 du \end{aligned}$$

Using identification between the last equation and equation 10, we deduce that $h_u = \left(\frac{\partial X_u}{\partial F}, \frac{\partial X_u}{\partial Y} \right)$, and then that $\phi_F = \frac{\partial X_u}{\partial F}$ and $\phi_Y = \frac{\partial X_u}{\partial Y}$. It can easily be checked that $d\phi_u = dX_u$ because X^* is a martingale under the measure \mathbb{P}^* . The portfolio ϕ is then self-financing.

Using this self-financing strategy, the insurer can manage the derivative contract X^j . If assumptions are acceptable, the financial strategy error depends on the frequency where futures positions of the replicating portfolio are adjusted.

3 TESTS

This subsection aims at illustrating the theoretical results with tests of the strategy management. Tests are realized for each step of the insurance management strategy. The first subsection presents the test of the premium of the instantaneous insurance contract and the second presents the test of the derivative strategy management.

3.1 Test of the premium of the instantaneous insurance contract

This subsection aims at testing the second strategy step and is split into three parts. The first one describes the used data and the second one proposes the tests. The third and last subsection comments the theoretical results and the tests.

3.1.1 The crop yield data of Illinois State

The used county yield historical data of Illinois State during 1972–2002 are public data of the National Agricultural Statistics Service.⁶ The Illinois State includes 102 counties. For each year, we know the aggregate crop yield and the cumulative planted acre of each county.

The yield data were adjusted for trends to reflect the 2002 production levels. The rate, obtained with an exponential regression of Illinois state data during 1973–2002 is 1.22%. The crop average yield of Illinois producers is 144.95 bushels per acre with an annual coefficient of variation of 13.95% (a minimum of 86.48 and a maximum of 171.68). Data include extreme events as the low yield crop of the years 1983 and 1988.

The range correlation between the county crop yield noted Y_{it} and the state crop yield during 1973–2002 is $[0.51, 0.94]$ with a mean of 0.84. Therefore, the county crop yield risk includes systemic and idiosyncratic risk components.

Of course, the perfect conditions to test the first step of the insurer management strategy need a set of farm crop yield data. However, the county yield data let us carry out a significant test of the management strategy because the county crop yield risk includes, as farm risk, systemic and idiosyncratic components.

3.1.2 Pooling tests description

Because we used county yield data, the tested contract is similar to the Group Risk Plan and the Group Risk Income Plan. The loss of the county i is $\ell_i = F_T \times \max(y_m^i - Y_{it})$ and the indemnity is defined by $I(\ell_i) = \ell_i$ because λ is chosen equal to 1. The county parameters α_i , β_i and γ_i of Equation 6 are estimated using the ten previous years' values. According to our assumption (normal distribution of ζ), we use least square regression for estimation. The minimum yield Y_{im} of the county depends on these historical results and a unique value Y_m using the relation:

$$Y_{im} = \alpha_i + \beta_i Y_m \quad (11)$$

Therefore, the premium of the instantaneous contract $g(F_T, Y_T, i)$ can be computed for each county for each year. According to our assumptions, the insured area ratio is uniform throughout the State. Because of the high variability of the instantaneous contract premium, the usual loss ratio (Indemnities/Premiums) is unadapted. Therefore, we test if the error ξ of the instantaneous contract management is near to 0 each year, where:

$$\xi = \text{Indemnities} - \text{Premiums} = \sum_{i=1}^{102} a_i F \times \max(Y_{im} - y_i, 0) - \sum_{i=1}^{102} a_i g(F, Y, i) \quad (12)$$

where a_i is the planted corn area ratio of the county i . Because we need the ten previous years values to define the parameters, all results concern the period 1982–2002.

3.1.3 Instantaneous insurance contract management tests results

Let us specify that the indemnity variabilities are 37.57, 28.89 and 21.33 where Y_m are equal respectively to 135, 125 and 115 (bu/acre). Knowing that average indemnities per acre are respectively \$19.28, \$13.06 and \$8.73, it results that the indemnity coefficients of variation are respectively 195%, 221% and 244%. As presented by Miranda and Glauber (1997), the percentage of variability observed for conventional insurance lines is very lower (e.g. Auto collision: 6%, Workers compensation: 9%, Crop hail: 15%).⁷ Moreover, it is interesting to relate this value to the concept of solvency margin. The legal safety minimum of the solvency margin is around 15% to 20% of premium in most of European Countries. Then, we clearly deduce that traditional private insurance is not able to support this high variability.

However, the standard deviation of error ξ of the instantaneous contract is significantly smaller than the indemnity variability. They are respectively 3.34, 3.48 and 3.43 (\$/acre), then they represent 17%, 27% and 39% of the average indemnities. This results are still higher than variability coefficients of conventional insurance lines but this values are now close. Moreover, let us precise that double static hedging of Mason et al. (2003), mentioned in introduction, reduces the standard deviation around by half. The derivative X^i are really more effective.

The errors are illustrated in figure 3 when $Y_m = 125$ (the error is null if the point is on the bisecting line). Errors are really small if we consider that on the one hand we only used basic assumptions and, on the other, the tested period includes extreme events.⁸

3.2 Test of the derivative strategy management

We use for the test of the first step, the corn yield futures quotation data during 1997-1998. This subsection aims at testing the financial management step of the crop yield contract. First, it describes the Chicago Board of Trade quotations data used. It then proposes the tests and, finally, presents and comments the tests results.

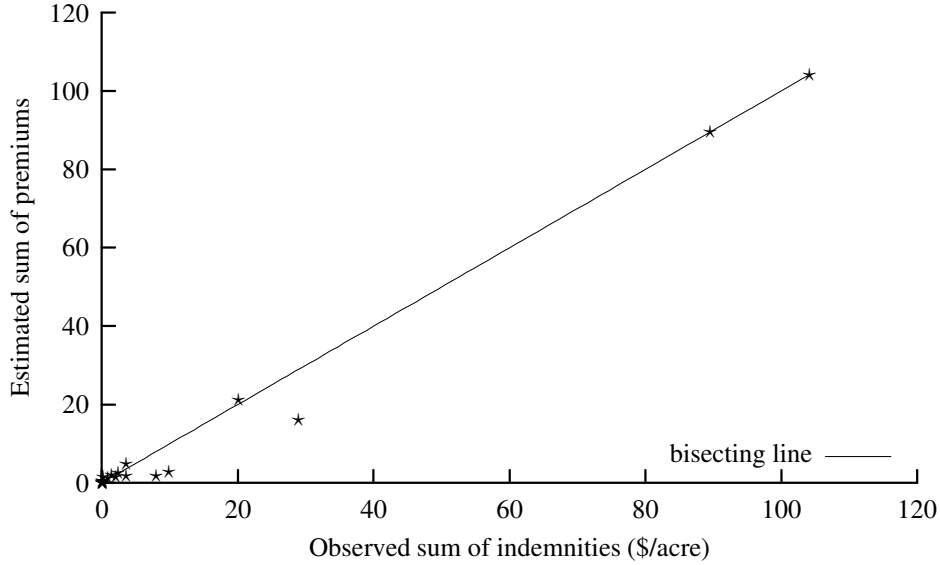


Figure 3: The instantaneous contract estimated premium relative to the observed value when $Y_m = 125$ (1982–2002).

3.2.1 The CBOT quotation data

The data of the Chicago Board of Trade used are quotations of the corn price futures and the Illinois crop yield futures during 1995–2000. For the crop yield futures, we only take into account the settlement of January 1997 and January 1998 because more liquidity has been founded (respectively 42 and 140 contracts exchanged). The corresponding corn price futures are respectively December 1996 and December 1997. Moreover, the risk-free rate of 1996 used is 5,20% and that of 1997 is 5,14% (From Econstat, US Treasury Instrument).

3.2.2 The financial management tests description

The test uses exactly the theoretical strategies presented in the previous section. Futures positions are adjusted each quotation day (at closing price). To get close to real conditions, the geometric Brownian motions parameters of futures contracts are estimated on the previous year quotation using the classical method presented by Hull (2000). The covariance between the two Brownian motions is estimated by the instantaneous correlation between F_t and Y_t . For the 19 December 1996 settlement test, we used the following parameters: $\mu_F = 0,07$, $\sigma_F = 0,268$, $\mu_Y = -0,06$, $\sigma_Y = 0,0556$, $\rho = -76,68\%$ and $r = 5,07\%$. For the 19 December 1997 settlement test, we used: $\mu_F = -0,082$, $\sigma_F = 0,244$, $\mu_Y = -0,019$, $\sigma_Y = 0,0580$, $\rho = -93,15\%$ and $r = 5,02\%$.

We choose to test the derivative contract management in the case of the Champaign county of Illinois. These parameters of the crop yield hazard are estimated in the previous subsection. In 1996, the estimation gives $\alpha_i = -22.67$, $\beta_i = 1.22$ and $\gamma_i = 14.39$, and in 1997 $\alpha_i = -23.71$, $\beta_i = 1.23$ and $\gamma_i = 14.39$.

As in the instantaneous contract management test, we examine three different contracts. $\lambda = 100\%$ for each contract and Y_m are respectively 135, 125 and 115 bu/acre. Using Campaign parameters, we deduce that Y_{im} are respectively 141.88, 129.69 and 117.50 bu/acre in 1996 and respectively 142.28, 129.98 and 119.69 bu/acre in 1997.

Moreover, an explicit formula of the derivative price is not available. Then, the price X_t must be estimated:

$$\begin{aligned} X_t &= e^{rt} \mathbb{E}_{\mathbb{P}^*} [X^*] \\ &= e^{-r(T-t)} \mathbb{E}_{\mathbb{P}^*} [F_T \lambda ((y_m^j - \alpha_j - \beta_j Y_T) (N(\eta^j) - N(\kappa^j)) + \gamma_j (f(\eta^j) - f(\kappa^j)))] \end{aligned}$$

Let us introduce a control variate $E(F_T, Y_T) = F_T \times \max(Y_{im} - \alpha_i - \beta_i Y_T, 0)$ or $\beta_i F_T \times \max(Y_m - Y_T, 0)$. First, we note that $Pr^j(F_T, Y_T) \geq E(F_T, Y_T)$ and that $Pr^j(F_T, Y_T) \approx E(F_T, Y_T)$ for extreme values of Y_T . Secondly, we recognize that E is the difference between two geometric Brownian motions and then E could be assimilated with an exchange option. More precisely, $E(F_T, Y_T)$ is equal in law to $\max(S_T^1 - S_T^2)$ when S^1 and S^2 are two geometric Brownian motions with: $S_T^1 = \beta_i F_T Y_m$, $S_T^2 = \beta_i F_T Y_T$, $\sigma_1 = \sigma_F$, $\sigma_2 = \sqrt{\sigma_F^2 + \sigma_Y^2 + 2\delta\sigma_F\sigma_Y}$ and the correlation coefficient:

$$\delta_S = \frac{\sigma_F^2 + \delta\sigma_F\sigma_Y}{\sigma_1\sigma_2}$$

As proved by Guinvarc'h et al. in the same framework, it results under \mathbb{P}^* that $S_t^1 = \beta_i F_t Y_m$ and that $S_t^2 = \beta_i \times \exp((r + \delta\sigma_F\sigma_Y)(T - t)) \times F_t Y_t$. Thirdly, the price of this exchange option is known by the Margrabe formula. Also, we deduce that E is a suitable control variate to estimate $Pr^j(F_T, Y_T)$. Then, we would like to estimate the value of the replicating portfolio ϕ_u . We note that:

$$\frac{\partial X_u}{\partial F} = \frac{\partial X_u}{\partial X_u^*} \frac{\partial X_u^*}{\partial F_u^*} \frac{\partial F_u^*}{\partial F_u} = e^{ru} \frac{\partial X_u^*}{\partial F_u^*} e^{-ru} = \frac{\partial X_u^*}{\partial F_u^*}$$

Next, the Fubini theorem gives:

$$\frac{\partial X_u}{\partial F} = \frac{\partial}{\partial F_u^*} \mathbb{E}_{\mathbb{P}^*} [X_T^*] = \mathbb{E}_{\mathbb{P}^*} \left[\frac{\partial}{\partial F_u^*} X_T^* \right]$$

Using the definition of $Pr^j(F_T, Y_T)$, we deduce that $\frac{\partial X_u}{\partial F} = \frac{X_u}{F_u}$. Moreover, knowing that $\frac{\partial}{\partial Y_u} Pr^j(F_T, Y_T) = -F_T \lambda \beta_i (N(\eta_i) - N(\kappa_i))$, we also obtain that:

$$\frac{\partial X_u}{\partial Y} = -\mathbb{E}_{\mathbb{P}^*} [e^{-r(T-t)} \times F_T \lambda \beta_i (N(\eta_i) - N(\kappa_i))]$$

Therefore, we are able to estimate the portfolio ϕ_u used in the risk management strategy of the crop yield insurance contract.

Y_m	Dec 1996			Dec 1997		
	135	125	115	135	125	115
Beginning Price	13.04	3.83	0.73	21.48	7.91	1.99
Settlement Price	12.13	3.03	0.45	32.56	12.97	3.38
Strategy Results	13.69	3.56	0.44	34.07	13.59	3.44
Error	-1.56	-0.53	0.01	-1.51	-0.62	-0.06
Min X_t	11.58	2.91	0.44	2.71	0.46	0.05
Max X_t	52.58	22.71	6.78	38.71	16.62	4.87

Table 1: Financial management strategy tests results.

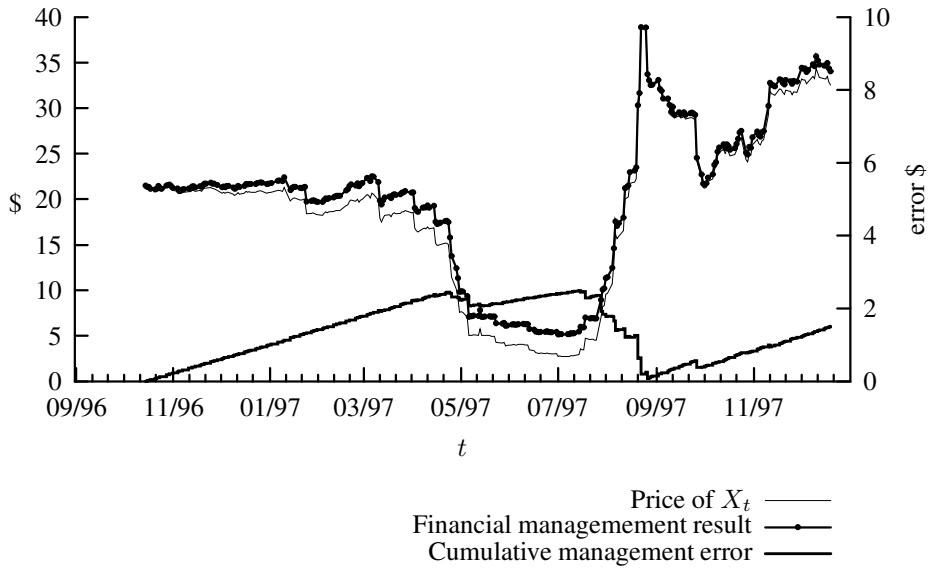


Figure 4: Illustration of the financial management strategy test when $Y_m = 135(\text{bu}/\text{acre})$.

Financial management tests results and comments

Table 1 presents the tests results. In spite of the low liquidity of the crop yield futures, we obtain a low error for the management strategy. The financial management strategy stated in the theoretical section is then performed to manage a portfolio of crop yield insurance contract.

This strategy is illustrated in figure 4 with the Champaign parameters when Y_m is chosen at 135. The value of X_t reaches a minimum of \$2.71 and a maximum of \$38.71. Then, the figure shows that management strategy is suitable to manage the large variations of X_t . The beginning value of derivative (X_0) is \$21.48 and this settlement value is \$32.56. The result of the financial management strategy gives \$34.07. Therefore, the settlement error is equal to \$-1.51. In respect of the volatility of the derivative price, errors are acceptable. Both the instantaneous insurance premium tests and the financial tests prove the practical ability for an insurer to manage a crop yield insurance contract.

4 CONCLUSION

Can a private insurer manage an agricultural crop insurance contract that includes a large systemic risk component? As a conclusion to this paper, it is in theory feasible and in practice manageable under the existence of futures contracts.

A strong implication of the above model is the need for insurance companies to have access to financial markets. Today, the worlds of finance and insurance are tightly linked so they are technically able to unite their expertise. However, there is a legal issue as regulations in many countries usually limit the capacity of actors to combine their expertise.

The strategy proposed in this paper opens commercial prospects for insurers. Furthermore, it could open opportunities for financial markets to include new contracts. If this model offers interesting prospects, it has some limits. The main limit of this work is market completeness. Our model assumes the existence of the price futures contract and the crop yield futures contract. The Chicago Board of Trade quoted crop yield contracts from 1995 to 2000. The contract was removed due to market imbalance and therefore a lack of liquidity. The Exchange has been working on many contract improvements in order to reopen this market for different agricultural crops. The model of a crop insurance contract, as developed in this paper, could bring an additional and important liquidity to this incoming market.

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NOTES

¹Two years later, Li and Vukina measure the effectiveness of this hedging for North Carolina producers.

²According to this principle, the insured cannot profit from loss.

³Abundant literature deals with a more adapted financial model that includes stochastic volatility and/or jump processes. Nevertheless, this assumption is the standard of continuous time financial models proposed by Black and Scholes and it may enable us to prove the interest of our insurance risk management approach.

⁴ \mathbb{P}^* is the unique measure equivalent to \mathbb{P} where $\begin{pmatrix} F_t^* \\ Y_t^* \end{pmatrix}$ is a local martingale under \mathbb{P}^* .

⁵As shown by Ramaswami and Roe (2004), $(\zeta_j)_{j=1,\dots,n}$ are not independent. Then, we cannot apply the central limit theorem to prove the convergence in probability of the error pooling.

⁶www.usda.gov/nass/

⁷US statistics during 1963-1992.

⁸In 1988 for example, indemnities reached 36% of the possible maximum indemnity estimated at \$350 per acre ($\sum_{i=1}^{102} a_i F \times Y_{im}$).