<u>Computational cross evaluation of financial and demographic components in the actuarial framework</u>

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Abstract

The aim of the study is to perform a compared sensibility analysis of the effects produced by the modification of financial (interest and/or actualisation rates) and demographic technical basis (longevity tables) for actuarial issues such as the ones faced by public or private pension funds. The results will be synthesised in terms of cross elasticity among the different existing components.

This study becomes particularly interesting with reference to the “longevity risk” issue, meaning the risk that the (private or public) insurer has to face as a combined consequence of the mortality reduction and the change in the interest rate of the investments done in order to cover the technical reserves.

The creation and the use of projected tables that are able to estimate the future survival trends, together with the need to take into account the future financial scenarios, imply a serious calculation complexity that requires the use of computational environments able to model the problem and to perform the production of synthetic indexes measuring the risk of the insurer.

The paper will analyse the problem using the stochastic approach, both from the financial and demographic sides, and will completed through the use of adequate computational environments (Apl2, J , Matlab, …) able to manage array data structures and the related synthetic symbolism developed by the authors.

Background

The Law 335/95 has deeply renewed the structure of the Italian pension structure, by substituting the system based on the link to the pre-retirement salaries with the one based on the actual contribution to the pension fund during the working life and the consequent post-retirement distribution based on transformation coefficients.

Analytically, these transformation coefficients correspond to the ratio between the first yearly pension instalment and the accrued fund; this fund is equal to the sum of worker periodic contribution capitalised according to a revenue rate equal to the last 5 years mean of GDP variation rate. Law 335/95 foresees the timing and the criteria for the revision and update of transformation coefficients, stating that these have to be re-calculated every ten years "on the base of demographic surveys and the actual trend of long term GDP".

The formula for the calculation of transformation coefficients adopts the usual actuarial techniques for the coverage of old age and surviving dependants risks with some specification related to the transition from the old to the new system as described in the first paragraph (i.e. the unicity of transformation coefficients in relation to gender).

The parameters used for building the transformation coefficients can be distinguished in two categories: demographic and regulative. The main demographic parameters are the surviving probability, the probability to leave surviving dependants together with the age difference between the insured person and his/her surviving dependants (always according to age and gender). The most important regulative parameters are retirement age, the reversibility rate, the inverse relation of this with the surviving dependants income and the differential between the yield rate (the last 5 years mean of GDP variation rate) and the actualisation rate.

The model for the calculation of transformation coefficients

By indicating with

- $\Delta_x = \textit{transformation coefficient for retirement age}$
- $l_{x,s} = \textit{survival table of retired per age and gender}$
- $l^v_{x,\hat{s}} = \textit{survival table of surviving dependant per age and gender}$
- $s = \textit{gender} \qquad \hat{s} = \textit{gender of surviving dependant}$
- $mf_s = \textit{gender belonging frequency}$
- $x = \textit{retirement age}$
- $\omega = \textit{max age in survival tables}$
- $\Theta_{x+t,s} = \textit{probability of leaving surviving dependants for a person aged } x+t$
- $\eta = \textit{reversibility rate}$
- $\delta_s = \%\ \textit{of reversibility rate reduction according to income}$
- $\varepsilon_s = \textit{age difference between insured and surviving dependant}$
- $k = \textit{correction factor}$
- $r = \textit{yield rate}$
- $\sigma = \textit{services indexing rate}$

The calculation model of transformation coefficients can be summarised in the following formula:

$$\Delta_x = \frac{1}{\displaystyle\sum_{s=m,f} \frac{mf_s}{l_{x,s}} \sum_{t=0}^{\omega-x} \left(l_{x+t,s} \left(\frac{1+r}{1+\sigma}\right)^{-t} + \eta\delta_s \frac{l_{x+t,s} - l_{x+t+1,s}}{l^v_{x+t+1-\varepsilon_s,\hat{s}}} \Theta_{x+t,s} \sum_{\tau=1}^{\omega-x-t+\varepsilon_s} l^v_{x+t+\tau-\varepsilon_s,\hat{s}} \left(\frac{1+r}{1+\sigma}\right)^{-(t+\tau)} \right) - k}$$

Applying which, the above mentioned Law 335/95 provided the following results:

Age	57	58	59	60	61	62	63	64	65
Δ_x	4.72%	4.86%	5.01%	5.17%	5.34%	5.52%	5.71%	5.92%	6.14%

Cross evaluation of financial and demographic components

The previous calculation has been performed by using the following technical bases:

- Survival table 1990 (source: ISTAT):

$$l_{x,s}$$

- Probability of leaving surviving dependants 1989 (source: INPS):

$$\Theta_{x+t,s}$$

- Gender belonging frequency:

$$mf_m = mf_f = \frac{1}{2}$$

- Retirement age:

$$x = 57, 58, ..., 65$$

- Reversibility rate:

$$\eta = 60\%$$

- % of reversibility rate reduction according to income:

$$\delta_s = \begin{cases} 90\% & s = m \\ 70\% & s = f \end{cases}$$

- Age difference between insured and surviving dependant:

$$\varepsilon_s = \begin{cases} +3 & s = m \\ -3 & s = f \end{cases}$$

- Correction factor:

$$k = 0.423$$

- Net actualisation rate:

$$\frac{r - \sigma}{1 + \sigma} = 1.50\%$$

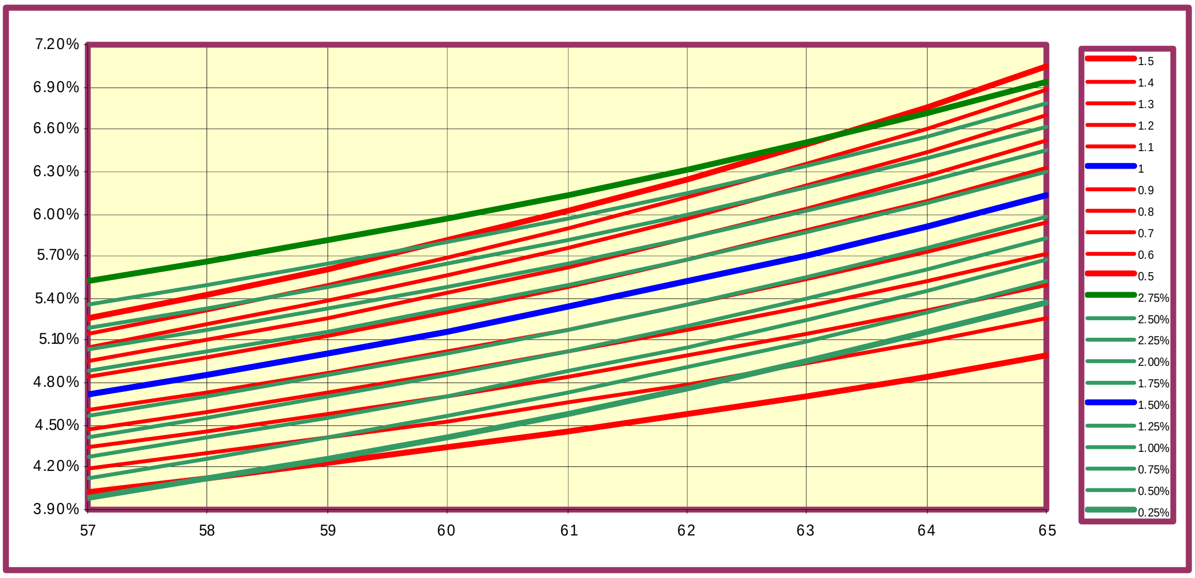
By repeating the calculation of transformation coefficients (considering the elimination probabilities modified through multiplying coefficients between 1.5 and 0.5) we obtain the following table in which it is possible to quantify the effect of reduction/extension of life in terms of increase/decrease of the corresponding transformation coefficients

Coeff	1.5	1.4	1.3	1.2	1.1	1	0.9	0.8	0.7	0.6	0.5
57	5.25%	5.15%	5.05%	4.95%	4.84%	4.72%	4.60%	4.47%	4.34%	4.19%	4.03%
58	5.43%	5.32%	5.21%	5.10%	4.98%	4.86%	4.73%	4.60%	4.45%	4.30%	4.13%
59	5.61%	5.50%	5.38%	5.26%	5.14%	5.01%	4.87%	4.73%	4.58%	4.41%	4.23%
60	5.81%	5.69%	5.57%	5.44%	5.31%	5.17%	5.02%	4.87%	4.71%	4.53%	4.34%
61	6.02%	5.89%	5.76%	5.62%	5.48%	5.34%	5.18%	5.02%	4.84%	4.66%	4.46%
62	6.25%	6.11%	5.97%	5.83%	5.67%	5.52%	5.35%	5.18%	4.99%	4.79%	4.58%
63	6.50%	6.35%	6.20%	6.04%	5.88%	5.71%	5.53%	5.35%	5.15%	4.94%	4.71%
64	6.76%	6.60%	6.44%	6.27%	6.10%	5.92%	5.73%	5.53%	5.32%	5.09%	4.85%
65	7.05%	6.88%	6.70%	6.52%	6.33%	6.14%	5.93%	5.72%	5.50%	5.26%	5.00%
Media	6.07%	5.94%	5.81%	5.67%	5.53%	5.37%	5.22%	5.05%	4.87%	4.68%	4.48%

and similarly, considering net actualisation rate between 2.75% and 0.25%, it is possible to obtain the following table in which the effect of rate increase/decrease can be measured in terms of increase/decrease of the corresponding transformation coefficients.

Tassi	2.75%	2.50%	2.25%	2.00%	1.75%	1.50%	1.25%	1.00%	0.75%	0.50%	0.25%
57	5.52%	5.36%	5.20%	5.04%	4.88%	4.72%	4.57%	4.42%	4.27%	4.12%	3.98%
58	5.66%	5.50%	5.33%	5.18%	5.02%	4.86%	4.71%	4.56%	4.41%	4.26%	4.12%
59	5.81%	5.64%	5.48%	5.32%	5.17%	5.01%	4.86%	4.70%	4.55%	4.41%	4.26%
60	5.97%	5.80%	5.64%	5.48%	5.32%	5.17%	5.01%	4.86%	4.71%	4.56%	4.42%
61	6.13%	5.97%	5.81%	5.65%	5.49%	5.34%	5.18%	5.03%	4.88%	4.73%	4.58%
62	6.32%	6.15%	5.99%	5.83%	5.67%	5.52%	5.36%	5.21%	5.06%	4.91%	4.76%
63	6.51%	6.35%	6.18%	6.02%	5.87%	5.71%	5.55%	5.40%	5.25%	5.10%	4.95%
64	6.72%	6.55%	6.39%	6.23%	6.07%	5.92%	5.76%	5.61%	5.45%	5.30%	5.16%
65	6.94%	6.78%	6.62%	6.45%	6.30%	6.14%	5.98%	5.83%	5.67%	5.52%	5.37%
Media	6.17%	6.01%	5.85%	5.69%	5.53%	5.37%	5.22%	5.07%	4.92%	4.77%	4.62%

These effects can be compared in the following chart



Through the calculation of the elasticity degree of transformation coefficients in relation to the variation of elimination probability and actualisation rate, by using the technical basis of Law 335/95, we obtain the following results:

Coeff	1.5	1.4	1.3	1.2	1.1	1	0.9	0.8	0.7	0.6	0.5
57	22.5%	22.9%	23.3%	23.8%	24.3%	24.9%	25.6%	26.3%	27.2%	28.2%	29.3%
58	23.2%	23.6%	24.1%	24.6%	25.1%	25.7%	26.4%	27.2%	28.0%	29.0%	30.2%
59	24.0%	24.4%	24.9%	25.4%	25.9%	26.6%	27.2%	28.0%	28.9%	29.9%	31.1%
60	24.8%	25.3%	25.7%	26.2%	26.8%	27.5%	28.1%	28.9%	29.8%	30.8%	32.0%
61	25.7%	26.1%	26.6%	27.1%	27.7%	28.4%	29.0%	29.8%	30.7%	31.8%	32.9%
62	26.6%	27.1%	27.5%	28.1%	28.6%	29.3%	30.0%	30.8%	31.7%	32.8%	33.9%
63	27.6%	28.0%	28.5%	29.0%	29.6%	30.3%	31.0%	31.8%	32.7%	33.8%	35.0%
64	28.6%	29.0%	29.5%	30.0%	30.6%	31.3%	32.0%	32.8%	33.8%	34.8%	36.1%
65	29.6%	30.1%	30.6%	31.1%	31.7%	32.4%	33.1%	33.9%	34.9%	35.9%	37.2%
Media	26.0%	26.5%	26.9%	27.5%	28.0%	28.7%	29.4%	30.2%	31.1%	32.1%	33.3%

Tassi	2.75%	2.50%	2.25%	2.00%	1.75%	1.50%	1.25%	1.00%	0.75%	0.50%	0.25%
57	20.3%	20.2%	20.1%	19.9%	19.8%	19.6%	19.5%	19.4%	19.2%	19.1%	18.9%
58	19.7%	19.6%	19.5%	19.4%	19.2%	19.1%	19.0%	18.8%	18.7%	18.6%	18.4%
59	19.1%	19.0%	18.9%	18.8%	18.7%	18.6%	18.4%	18.3%	18.2%	18.0%	17.9%
60	18.6%	18.5%	18.3%	18.2%	18.1%	18.0%	17.9%	17.8%	17.7%	17.5%	17.4%
61	18.0%	17.9%	17.8%	17.7%	17.6%	17.5%	17.4%	17.3%	17.1%	17.0%	16.9%
62	17.4%	17.3%	17.2%	17.1%	17.0%	16.9%	16.8%	16.7%	16.6%	16.5%	16.4%
63	16.8%	16.7%	16.7%	16.6%	16.5%	16.4%	16.3%	16.2%	16.1%	16.0%	15.9%
64	16.3%	16.2%	16.1%	16.0%	15.9%	15.9%	15.8%	15.7%	15.6%	15.5%	15.4%
65	15.7%	15.6%	15.5%	15.5%	15.4%	15.3%	15.3%	15.2%	15.1%	15.0%	14.9%
Media	17.9%	17.8%	17.7%	17.6%	17.5%	17.4%	17.3%	17.1%	17.0%	16.9%	16.8%

that enable to state as follows:

- a variation of x% in the elimination probability determines a similar average variation of 0.287*x% of transformation coefficients
- a variation of y% in the actualisation rate determines a similar average variation of 0.174*y% of transformation coefficients

thus

- the same variation of transformation coefficients is obtained in presence of a similar %al variation of actualisation rates that is equal to 5/3 of the corresponding %al variation of the elimination probabilities;
- in presence of a %al variation of the elimination probabilities it is possible to maintain the pre-existing level of transformation coefficients through an opposite variation of actualisation rates, which corresponds, in average, to the 5/3 of the coefficients variation. For instance, if the elimination probabilities would suffer a decrease of 20% - determining a 6% coefficient decrease – this variation could be compensated with an 33% increase of the actualisation rate passing from 1.50% to 2% and so on.

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