ANALYSIS OF TRENDS IN MORTALITY NEAR OR DURING RETIREMENT FOR FOUR EUROPEAN COUNTRIES

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ABSTRACT

Life expectancy at birth has improved dramatically over the course of the twentieth century. Over this period the greatest improvements in mortality rates have been seen in progressively older ages, a feature which we refer to as "the ageing of mortality improvements”.

In this paper we considered two mortality models to capture the ageing of mortality improvements by the fitting of formulaic mortality curves to actual mortality in each calendar year over a selected period. The first method consisted of fitting a Logistic mortality curve to values of q_x by individual age. The second method consisted of fitting a Weibull distribution to the probability from birth of death at individual ages, i.e. $\frac{lx}{lo} q_x$.

We carried out separate examinations as to mortality experience from the Human Mortality Database (“HMD”) for four European countries: France, Italy, Spain, former West Germany and former East Germany. We have treated former West Germany and former East Germany as separate countries (and for convenience refer to them as such) as mortality experience for these two former countries was separately presented in the HMD over the investigation period.

Trend lines were fitted to the progression in those parameters underlying the formulaic mortality curves and projected to derive mortality curves in future calendar years. The financial implications of the results were illustrated by calculating annuity values based on these projections.

We compare the differences in projected future mortality between the various countries. We discuss the implications of the mortality projections in light of some of the views expressed by experts in the fields of demography and medicine as to the limitations that should be considered in forward projecting mortality based on historical trends.

KEYWORDS: Mortality projection models, Life expectancy, Weibull distribution, Logistic

INTRODUCTION AND CONTEXT

The twentieth century saw very dramatic reductions in mortality rates but this was most marked at different age groups at different periods in the century. In the first half of the century significant improvements in the treatment of infectious diseases resulted in the reductions being most significant for children and young adults. In sharp contrast, in the second half of the century significant reductions in deaths associated with cardiovascular disease resulted in the main improvements being seen for those in their fifties and over.

These trends lead the authors to believe that attempts to allow for improving longevity by applying an overall percentage reduction factor to mortality rates fail to capture an important feature of past experience. The methodologies discussed in this paper are intended to include an implied allowance for the ageing of mortality improvements.

The effect of these improvements has been that far more individuals are living to advanced ages. This has significant implications for the costs of social insurance and the relative size of the working and retired population. As experts in mortality, actuaries may be expected by non-actuaries to be well placed to answer fundamental questions about these trends. In particular they may assume that actuaries have a clear and accurate view as to the course of future mortality.

The misperception of the level of actuarial expertise in this area presents significant dangers for the profession. We must, in the authors' view:

- increase public understanding of what we can and, most importantly, what we can't do;
- ensure that we make the best use of available data and appropriate models to project mortality based on past trends; and
- work with medical experts, demographers and others to develop models combining their technical expertise with ours.

There is a long history of fitting mathematical models to mortality experience. In 1825 Gompertz, on examination of census data, noted an exponential rise in mortality rates after sexual maturity. He provided a physiological explanation for this observation as "the average exhaustion of man's power to avoid death gained in equal proportion in equal intervals of age." The formula he proposed was:

$$\text{Force of mortality at age } x,\ \mu_x = Bc^x\ .$$

The relationship does not provide a good fit with experience at younger ages, and Makeham added a constant term in 1867 to reflect, in part, differences in cause of death between those due to accident and those due to disease. Heligman and Pollard further refined this process by proposing an eight-parameter model in 1980 with components designed to model childhood illnesses and mortality associated with pregnancy.

The Gompertz and Makeham formulae become increasingly inaccurate at very advanced ages. Logistic formulae have been suggested by Kannisto (1994), Beard (1971), and Perks (1932) that slow the rate of increase in mortality at older ages.

A number of experts in this area have expressed the view that mortality experience at very old ages might show a plateau in the rate of mortality, and proposed theories to explain this feature. However, concerns over the credibility of the data and doubts about the accuracy of stated ages at very high ages, even at the population level, makes the identification of such patterns difficult. In the case of the Human Mortality Database, on which our work was based, mortality rates by individual age over the age of 80 in a given calendar year were fitted to the actual mortality experience by means of a logistic regression.

For actuaries, a particular interest of a mortality model is the extent that it provides a methodology for projecting future mortality rates. The multifactorial nature of mortality improvements means that past trends in mortality rates may not necessarily provide a good forecast of future trends. This is, however, also true of rates of interest, rates of policy discontinuance and expenses; yet, over a long period of time it has been accepted actuarial methodology to use past experience as a best estimate of the future unless there was good reason to assume a change.

The future projection of past trends provides a basis of projection that is entirely within actuarial expertise. Future projections based on an understanding of the future possible interactions between different diseases, and the interplay of treatment and risk factors, is not an actuarial skill. Such projections would need to be based on detailed examination of cohort-based populations and the development of diseases within those populations. The availability of such databases is limited, and require significant medical interpretation.

We must, however, ensure that the past experience is properly analysed and that the model selected for projecting the future is appropriate having regard to past experience. If appropriate it can be adjusted to allow for differences expected on the grounds of specified factors. As the parameters in the mortality models, discussed in this paper, have shown historical time dependency, such models provide a methodology for projecting future mortality rates that, in our view, should be considered by actuaries.

APPROACH TO MODELLING

The trend in future longevity is unknowable. This is a critically important fact which actuaries need to be prepared to say loudly and clearly.

The trend in future mortality which emerges in practice will depend, amongst other factors, upon future medical advances. The qualitative effects of some of these advances can be predicted from medical work already done, but significant further work needs to be done to quantify their likely age-related impact. Other medical advances may not yet be foreseen.

Even where medical advances are made, their impact on population mortality will depend upon government spending policies and competing requirements for healthcare expenditure. This is likely to become an increasingly important factor as the numbers of elderly increase and the number at (current) working ages decrease.

A further unknown is the risk that diseases or other factors may emerge that may increase mortality, such as virulent strains of influenza or the spread of antibiotic-resistant bacteria or the consequences of an obesity "epidemic".

Given these uncertainties, projecting future trends in mortality is fraught with difficulty.

In the authors' view there are three basic ways of projecting future mortality:

- **a.** The analysis of past trends in death rates and their forward projection, without taking into account medical considerations;
- **b.** Projections based on medical data at the specific condition level; and
- **c.** Projections based on overarching medical constraints, most obviously, an upper practical limit to future human life spans, although others may perhaps be envisaged.

Approach (a) is the approach that comes most naturally to actuaries, and this paper provides examples of it. A predictive model based solely on medical data, that is (b) in isolation, is possible in principle, but the authors regard it as being, at least, several years away in practice.

Option (c) could be very important, although its impact on calculating capital values of annuities at the present time will depend on the point at which future mortality improvements become limited. The authors discuss the possibility of such constraints in a subsequent section of the paper, but for the purposes of this paper have not adjusted the models to allow for the possibility of such constraints.

The authors have done work involving a combination of approaches (a) and (b) and see considerable merit in, and scope to extend, this approach. This is an application of the actuarial approach discussed above, of assuming continuation of trends observed in the past but adjusting for known differences.

The work in question related to the UK which has shown much higher rates of mortality improvement for those born in a certain period (the "Cohort Effect" as discussed in Working Paper 1 of the Continuous Mortality Investigation Bureau and much other UK actuarial literature). The authors are of the view that a significant component of the excess mortality improvement experienced by this generation is attributable to changes in the prevalence of smoking. They developed a model to quantify this component of improvement and used it to adjust the results produced by the CMI.

The authors consider that starting with trend based models and adjusting them with medical information for a small number of conditions which are major causes of mortality is likely to provide the best practical method of moving towards (b).

DATA SOURCES

We used the Human Mortality Database ('HMD') as our source for age-specific mortality rates for both sexes and for all four countries. The HMD website is: www.mortality.org. The HMD represents a collaborative project between the Department of Demography at the University of California, Berkeley, United States, and the Max Planck Institute for Demographic Research in Rostock, Germany.

The HMD contains information on 23 countries, consisting of data on births, deaths, population size, and exposure to risk, together with detailed descriptions on the sources of the data. The goal of the project is to adopt uniform procedures for each country in the collection of, and verification of, data, and in calculating death rates and life tables. The HMD notes that, in particular, the issue of age exaggeration is addressed in part by the derivation of population estimates at older ages through the death counts themselves, employing extinct cohort methods, as age reporting in death registration systems is assumed to be more reliable than in census counts or official population estimates.

MEASURES OF MORTALITY

The investigation period was taken to be 1960 to 1998. For each calendar year in the investigation period, we obtained or calculated the following mortality measures for each calendar year, country and sex:

Probability of death in the next year for a life aged *x*, ie q_x;

Probability at birth of death at age *x* ie $\frac{lx}{lo} q_x$.

For the avoidance of doubt when we talk about probabilities of death in a particular calendar year, we mean probabilities based on death rates at each age *for that calendar year*. Thus the value of $\frac{lx}{lo} q_x$ for 1970 is derived from the age specific death rates in 1970 applied as a classic period actuarial life table without allowance for improvements. Mortality improvements are added explicitly at a later stage.

DESCRIPTION OF MORTALITY MODELS

Method 1 – Logistic Model - Probability of death in the next year of life

The logistic model we used was of the form**:**

$$q_x = \frac{1}{1+\alpha\beta^x}$$

Parameters α and β were selected for each calendar year in the investigation period to fit the mortality experience by individual age. This process was repeated separately for males and females, and for each of the different countries considered in this paper. We applied the statistical package SPSS to investigate and select the most appropriate parameters to fit the data.

The trend in each parameter was then analysed separately by country and sex and a formulaic trend fitted. This fitted trend was then used to project a value for each parameter in each future calendar year. The parameters set for each future calendar year were input to the formulaic curve to derive the projected mortality curve for that year.

Method 2 – Weibull Distribution - Probability from birth of death at age x

The Weibull Distribution was developed by Dr Waloddi Weibull in 1937 and was first introduced in 1951 by his paper "A Statistical Distribution Function of Wide Applicability." The Weibull Distribution is widely used in the analyses of the reliability and the life of physical structures and numerous other applications.

The Weibull Distribution is determined by two parameters, *c* (the scale parameter) and γ (the shape parameter). Compared to the exponential distribution (a special case of Weibull where

$\gamma = 1$, the Weibull Distribution can have a failure rate (of a material or a structure) that varies. This makes the distribution more suitable for models of mortality.

The shape of the Weibull Distribution resembles the normal distribution with a right-skew and a tail that is lighter. These characteristics of the Weibull Distribution have allowed us to fit the distribution to the probability at birth of death at age x, ie $\frac{lx}{lo} q_x$ with some success.

The probability density function (equation 1) and the probability distribution function (equation 2) of the Weibull Distribution both contain two parameters, c and γ (Miller, 1999):

$$f(x) = c\gamma x^{\gamma-1} \exp(-cx^{\gamma}), \tag{1}$$

$$F(x) = 1 - \exp(-cx^{\gamma}). \tag{2}$$

In order to fit the Weibull model, we used the "Method of Percentiles" (Klugman, 1998) to estimate the parameters.

The Method of Percentiles enables two equations with two unknowns to be written. Specifically we equated the probability distribution function to solve for the parameters which equated two theoretical percentiles with those of the actual data. We used the 50th percentile and the 95th percentile to fit the curve, as we were principally interested in the accuracy of fit at the higher ages.

The following equations were used to estimate the parameters:

$$F(x_1) = 1 - exp\ (-cx_1^{\gamma}) = 0.5 \tag{3}$$

where x_1 is the actual age corresponding to the 50th percentile in the data set and where $F(x_1)$ is the value of the probability distribution function of the Weibull Distribution, and

$$F(x_2) = 1 - exp\ (-\ cx_2^{\gamma}) = 0.95, \tag{4}$$

where x_2 is the actual age corresponding to the 95th percentile in the data set and where $F(x_2)$ is the value of the probability distribution function of the Weibull Distribution.

By ratioing ln (equation (3) – 1) to ln (equation (4) – 1), the parameter c disappears, and parameter γ can be derived. Using parameter γ, one can then derive parameter c.

In our investigations of probability of death at a specified age in conjunction with the Weibull Distribution, for ease of typographical presentation we define parameter α to be equal to parameter γ, and parameter β to be equal to $c^{-1/\gamma}$.

Parameters α and β were selected for each calendar year in the investigation period to fit the mortality experience by individual age. This process was repeated separately for males and females, and for each of the different countries considered in this paper.

The trend in each parameter was then analysed separately by country and sex and an appropriate trend fitted. This fitted trend was then used to project a value for each parameter in each future calendar year. The parameters set for each future calendar year were input to the formulaic curve to derive the projected mortality curve for that year.

SELECTION OF PARAMETERS FOR MORTALITY MODELS

Method 1 – Logistic Model

Table 1 contains the values of the parameters that were required by the logistic model for each of the countries or former countries considered for male and female lives separately for a number of selected years, as well as the appropriateness of the logistic model by means of the R^2 value.

Table 1 - Parameters and analysis of logistic models for selected years for male and female lives

	Males			**Females**		
	1960	**1980**	**1999**	**1960**	**1980**	**1999**
France						
Parameter α	8037.50	8706.43	12434.57	39899.64	46705.66	45382.51
Parameter β	0.91697	0.91861	0.91911	0.90342	0.90685	0.91284
R^2 value	100%	99%	100%	99%	98%	98%
Italy						
Parameter α	11931.12	15506.43	42480.68	52741.59	75317.78	97903.35
Parameter β	0.91318	0.91048	0.90319	0.89949	0.89908	0.90181
R^2 value	100%	100%	100%	100%	100%	99%
Spain						
Parameter α	19928.58	20440.59	25661.31	61522.55	95502.11	141278.49
Parameter β	0.90617	0.90918	0.90937	0.89680	0.89713	0.89781
R^2 value	100%	100%	100%	99%	99%	99%
W. Germany						
Parameter α	14343.84	18542.88	26251.91	71790.65	62267.41	62313.28
Parameter β	0.90886	0.90720	0.90805	0.89353	0.90001	0.90539
R^2 value	100%	100%	100%	100%	99%	99%
E. Germany						
Parameter α	17815.85	19422.31	19019.46	65425.35	63863.42	99973.66
Parameter β	0.90647	0.90491	0.91101	0.89473	0.89638	0.89833
R^2 value	100%	100%	100%	99%	99%	99%

We considered a number of different categories of trend lines, including linear and exponential, for each of the parameters required by the logistic model. These were of the form:

Linear: Parameter α or β = C + D * (Calendar year - 1959)

Exponential: Parameter α or β = C * exp (D * (Calendar year – 1959))

Table 2 provides details on the values of C and D of the chosen trend line for each of the countries considered for males and females separately, as well as the appropriateness of the chosen trend line by means of the R^2 value.

Table 2 - Parameters and analysis of trend lines fitted to logistic models from 1960 to 1999 for male and female lives

	Males				Females			
	Trend	**C**	**D**	R^2	**Trend**	**C**	**D**	R^2
France								
Parameter α	Lin	7973.19	71.47	45%	Lin	42044.17	168.09	35%
Parameter β	Lin	0.91546	0.00013	58%	Lin	0.90184	0.00026	90%
Italy								
Parameter α	Exp	9958.85	0.03	87%	Lin	46974.78	1185.84	94%
Parameter β	Lin	0.91382	-0.00023	83%	Lin	0.89922	0.00006	39%
Spain								
Parameter α	Lin	20255.83	59.30	17%	Lin	53525.17	2043.46	94%
Parameter β	Lin	0.90509	0.00016	71%	Lin	0.89634	0.00004	19%
W. Germany								
Parameter α	Lin	14395.66	239.44	84%	Exp	58300.57	0.00	29%
Parameter β	Exp	0.90695	0.00003	3%	Lin	0.89541	0.00022	91%
E. Germany								
Parameter α	Exp	25442.48	-0.02	44%	Lin	64893.22	369.08	23%
Parameter β	Lin	0.90083	0.00029	59%	Lin	0.89350	0.00012	62%

Method 2 – Weibull distribution model

Table 3 provides the values of the parameters that were required by the Weibull distribution model for each of the countries or former countries considered for male and female lives separately for a number of selected years, as well as the appropriateness of the Weibull distribution model by means of the R^2 value.

Table 3 - Parameters and analysis of Weibull distribution models for selected years for male and female lives

	Males			Females		
	1960	1980	1999	1960	1980	1999
France						
Parameter α	6.89	7.17	8.20	8.89	9.96	11.12
Parameter β	75.46	77.90	82.14	81.50	85.16	88.67
R^2 value	95%	96%	99%	98%	99%	99%
Italy						
Parameter α	7.11	7.48	8.52	8.70	9.78	10.78
Parameter β	76.37	77.94	82.49	80.69	83.95	87.83
R^2 value	95%	98%	99%	98%	100%	99%
Spain						
Parameter α	7.23	7.81	8.40	8.14	9.68	11.18
Parameter β	76.49	79.67	82.10	80.76	84.91	87.85
R^2 value	94%	99%	99%	96%	99%	99%
W. Germany						
Parameter α	6.98	7.34	8.04	8.79	9.43	10.45
Parameter β	75.23	77.21	81.58	79.87	83.26	86.84
R^2 value	98%	98%	99%	98%	99%	99%
E. Germany						
Parameter α	7.24	7.54	7.85	9.09	9.43	10.20
Parameter β	75.57	75.95	80.30	79.74	81.04	86.18
R^2 value	97%	97%	98%	98%	99%	99%

We considered a number of different categories of trend lines, including linear and exponential, for each of the parameters required by the Weibull distribution model. These were of the form:

Linear: Parameter α or β = C + D * (Calendar year - 1959)

Exponential: Parameter α or β = C * exp (D * (Calendar year – 1959))

Table 4 provides details on the values of C and D of the chosen trend line for each of the countries or former countries considered for males and females separately, as well as the appropriateness of the chosen trend line by means of the R^2 value.

Table 4 - Parameters and analysis of trend lines fitted to Weibull distribution models from 1960 to 1999 for male and female lives

	Males				Females			
	Trend	**C**	**D**	R^2	**Trend**	**C**	**D**	R^2
France								
Parameter α	Lin	6.48	0.04	92%	Lin	8.63	0.06	98%
Parameter β	Lin	74.28	0.19	96%	Lin	81.04	0.20	99%
Italy								
Parameter α	Lin	6.74	0.04	94%	Lin	8.54	0.05	97%
Parameter β	Lin	75.22	0.17	94%	Lin	80.12	0.19	99%
Spain								
Parameter α	Lin	7.13	0.03	94%	Lin	7.98	0.08	99%
Parameter β	Lin	76.01	0.16	97%	Lin	80.32	0.20	98%
W. Germany								
Parameter α	Lin	6.68	0.03	93%	Lin	8.52	0.05	97%
Parameter β	Lin	74.05	0.17	93%	Lin	79.44	0.18	98%
E. Germany								
Parameter α	Lin	7.33	0.01	12%	Lin	9.02	0.02	65%
Parameter β	Lin	75.16	0.07	60%	Lin	79.08	0.13	82%

We illustrated the appropriateness of the choice of time-dependent parameters for both logistic and Weibull distribution models by means of annuities commencing in year 1960. We calculated the capital value of annuities payable continuously from different valuation ages at an interest rate of 4% per annum.

Table 5 sets out values for annuities assumed to be paid continuously for each country according to three bases. Basis 1 uses actual mortality experience over the period 1960 to 1999. Bases 2 and 3 use mortality rates as derived from logistic and Weibull distribution models respectively, with those models being constructed from parameters α and β taken from trend lines applying over the period 1960 to 1999. All bases assume that mortality rates applying in year 1999 remain constant for subsequent years.

Table 5 - Value of annuities commencing in year 1960 according to Bases 1, 2 and 3 for male and female lives

	Males			Females		
	Basis1	**Basis2**	**Basis3**	**Basis1**	**Basis2**	**Basis3**
France						
Age 55	12.9	13.1	12.7	15.6	15.9	15.6
Age 65	9.5	9.5	9.1	11.6	12.1	11.4
Age 75	6.2	6.1	6.1	7.4	8.0	7.3
Italy						
Age 55	13.1	13.1	12.9	15.3	15.5	15.2
Age 65	9.6	9.5	9.2	11.2	11.6	11.0
Age 75	6.3	6.1	6.1	7.0	7.5	7.0
Spain						
Age 55	13.5	13.6	13.2	15.4	15.6	15.2
Age 65	9.8	9.8	9.5	11.3	11.5	11.1
Age 75	6.2	6.2	6.2	7.2	7.2	7.2
W. Germany						
Age 55	12.7	12.9	12.5	14.9	15.1	14.9
Age 65	9.1	9.0	8.9	10.8	11.0	10.6
Age 75	6.0	5.6	5.8	6.7	6.9	6.7
E. Germany						
Age 55	12.7	12.9	12.6	14.5	14.6	14.5
Age 65	9.1	9.2	8.9	10.6	10.6	10.3
Age 75	5.9	5.6	5.7	6.5	6.6	6.4

FINANCIAL PROJECTIONS OF MORTALITY MODELS

We used the methods of projecting measures of mortality that we discussed in the previous sections to illustrate differences both in terms of country and choice of model. We calculated the capital value of annuities payable continuously from different valuation start dates and ages and an interest rate of 4% per annum, using methods 1 and 2 to project future measures of mortality.

Method 1 – Logistic model – male lives

Graph 1 sets out the capital value of annuities according to method 1 for all countries considered for males aged 65 with commencement dates from 1960 to 2050. Table 6 provides a summary of the capital value of the annuities for selected commencement dates.

Graph 1
Logistic Model : Male annuity values for age 65 from different commencement years

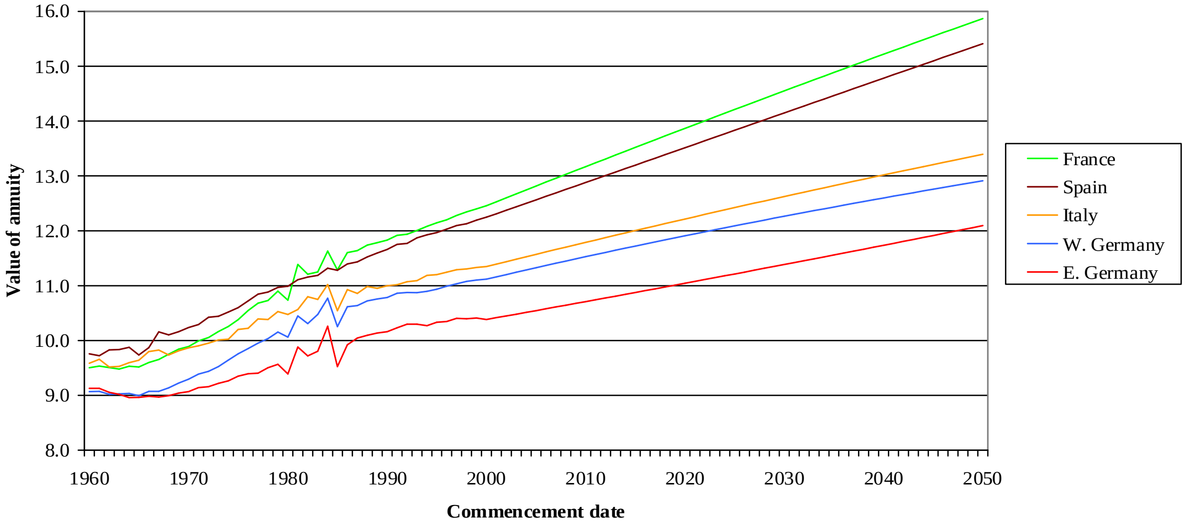
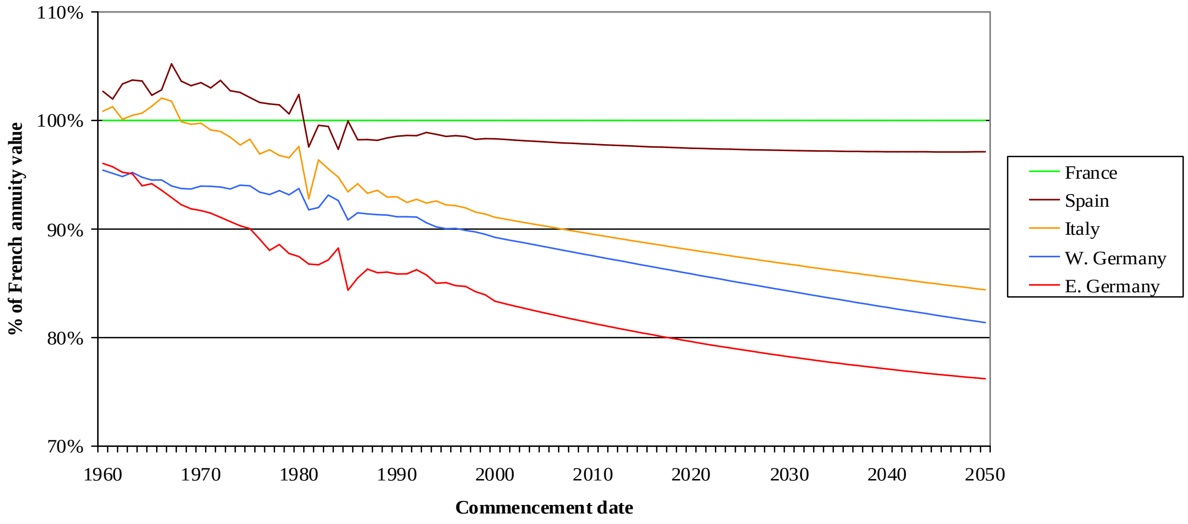


Table 6 - Value of annuities for male lives aged 65 in different countries and former countries for selected commencement dates – Logistic Model

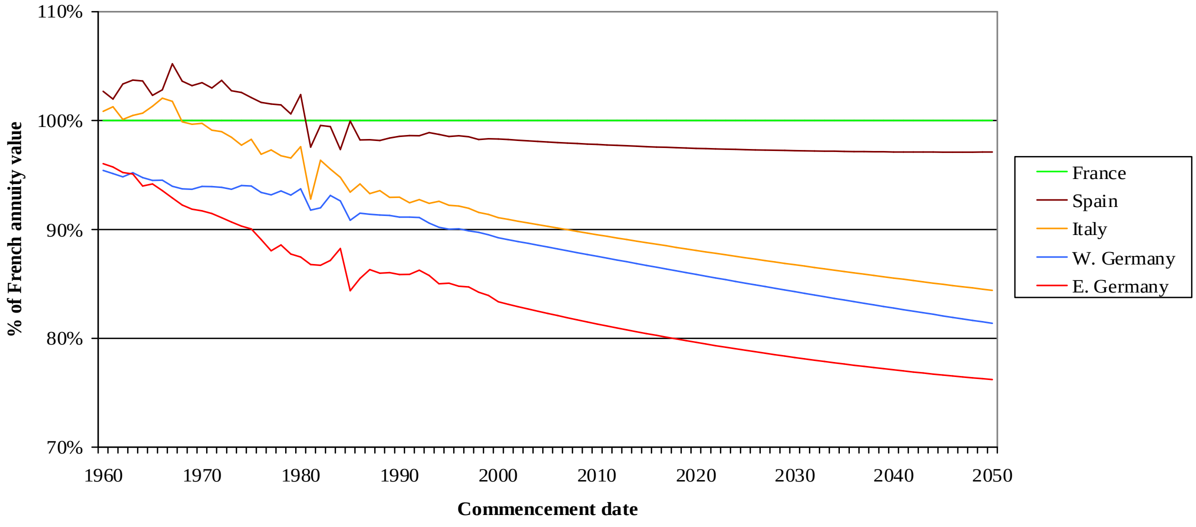
	1960	1980	2005	2020	2035	2050
France	9.50	10.73	12.82	13.87	14.89	15.87
Italy	9.58	10.48	11.57	12.21	12.82	13.39
Spain	9.76	10.99	12.56	13.51	14.47	15.41
W. Germany	9.07	10.06	11.33	11.91	12.43	12.91
E. Germany	9.13	9.39	10.55	11.04	11.56	12.09

We set out comparisons of the capital value of annuities for different countries and former countries in graphs 2, 3, and 4 relating to valuation ages of 55, 65 and 75 respectively. In each graph the comparison is with respect to the capital value of annuities for French males for each commencement year.

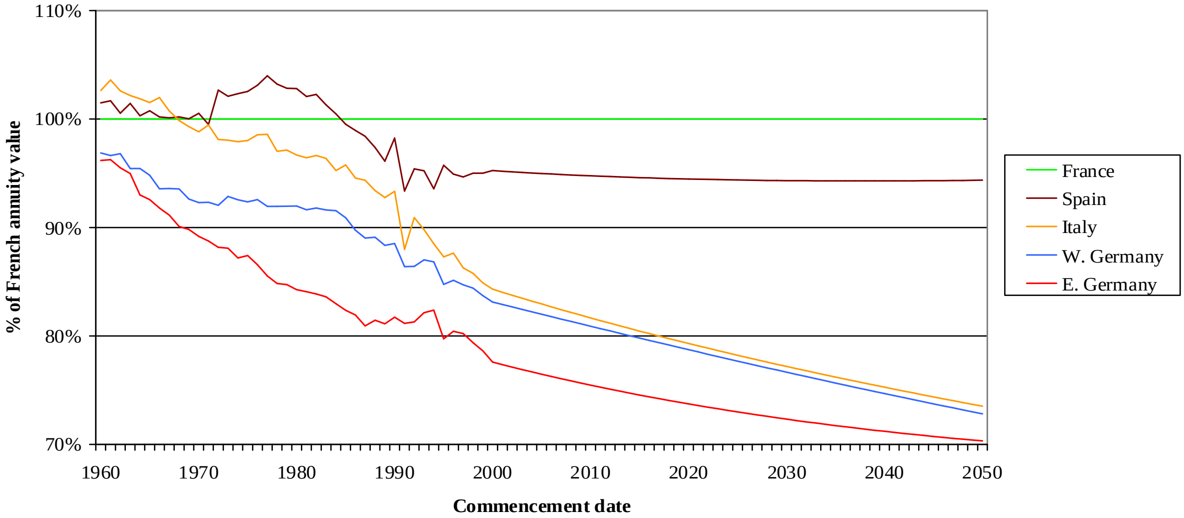
Graph 2
Logistic Model: Male annuity value comparisons for age 55 from different commencement years



Graph 3
Logistic Model: Male annuity value comparisons for age 65 from different commencement years



Graph 4
Logistic Model: Male annuity value comparisons for age 75 from different commencement years



Method 2 – Weibull distribution model – male lives

Graph 5 sets out the capital value of annuities according to method 2 for all countries considered for males aged 65 with commencement dates from 1960 to 2050. Table 7 provides a summary of the capital value of the annuities for selected commencement dates.

Graph 5
Weibull Model : Male annuity values for age 65 from different commencement years

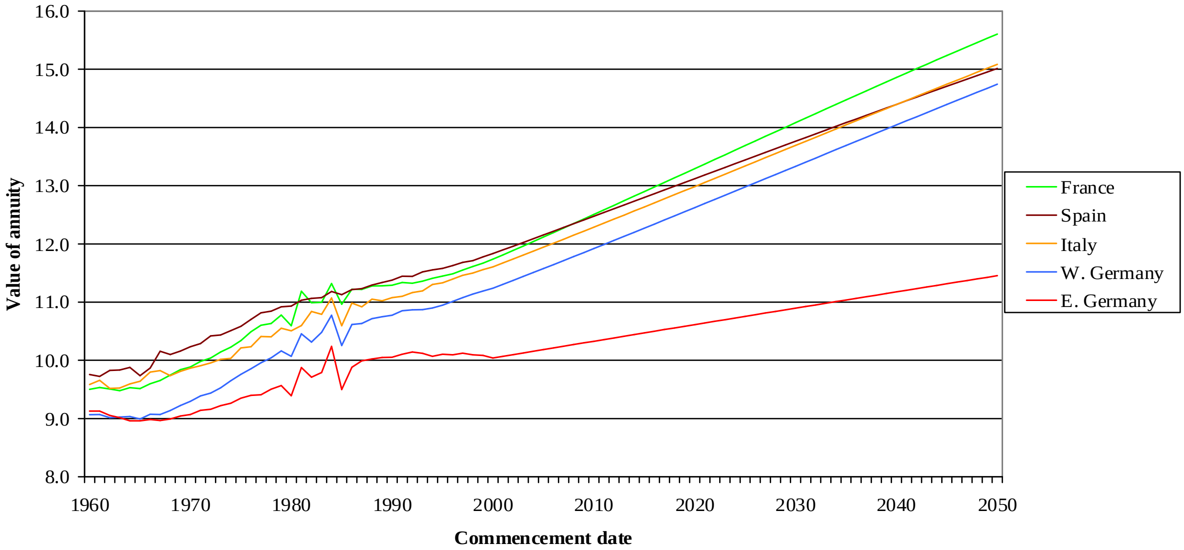
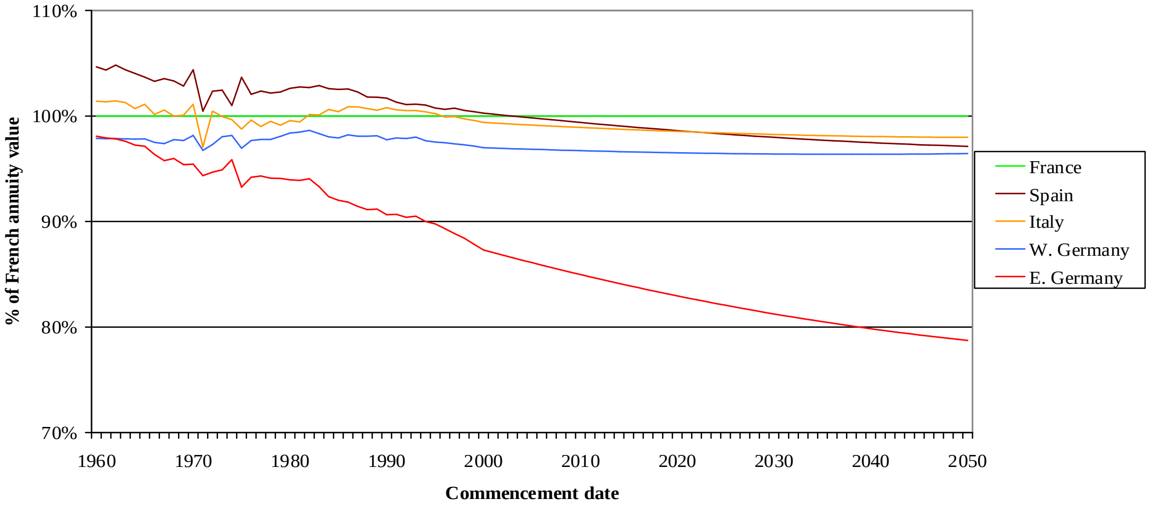


Table 7 - Value of annuities for male lives aged 65 in different countries and former countries for selected commencement dates – Weibull Model

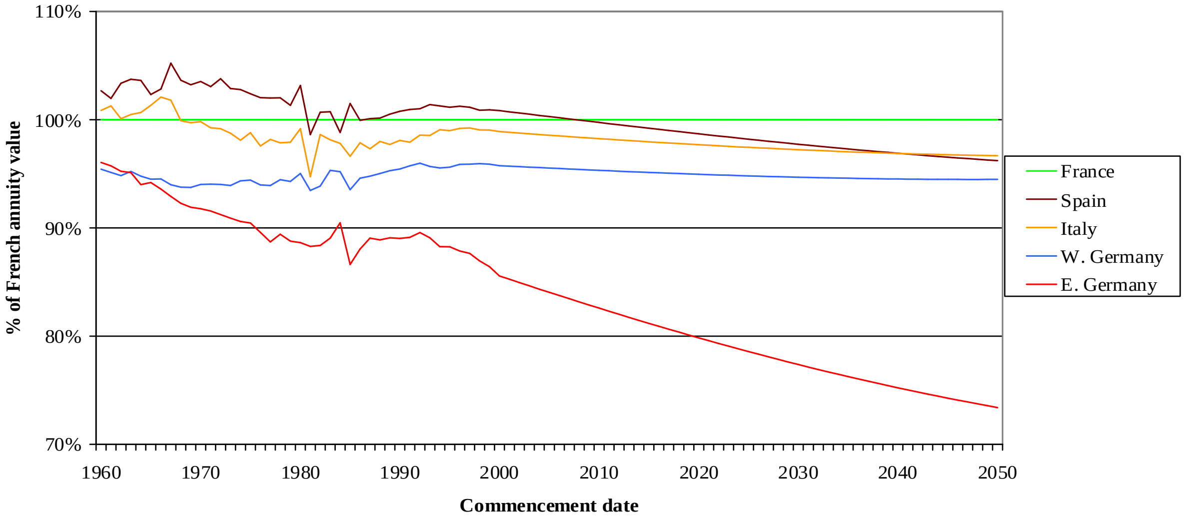
	1960	1980	2005	2020	2035	2050
France	9.50	10.59	12.12	13.29	14.47	15.60
Italy	9.58	10.51	11.94	12.99	14.04	15.08
Spain	9.76	10.93	12.15	13.12	14.08	15.01
W. Germany	9.07	10.07	11.58	12.62	13.69	14.74
E. Germany	9.13	9.39	10.18	10.61	11.04	11.45

We set out comparisons of the capital value of annuities for different countries in graphs 6, 7, and 8 relating to valuation ages of 55, 65 and 75 respectively. In each graph the comparison is with respect to the capital value of annuities for French males for each commencement year.

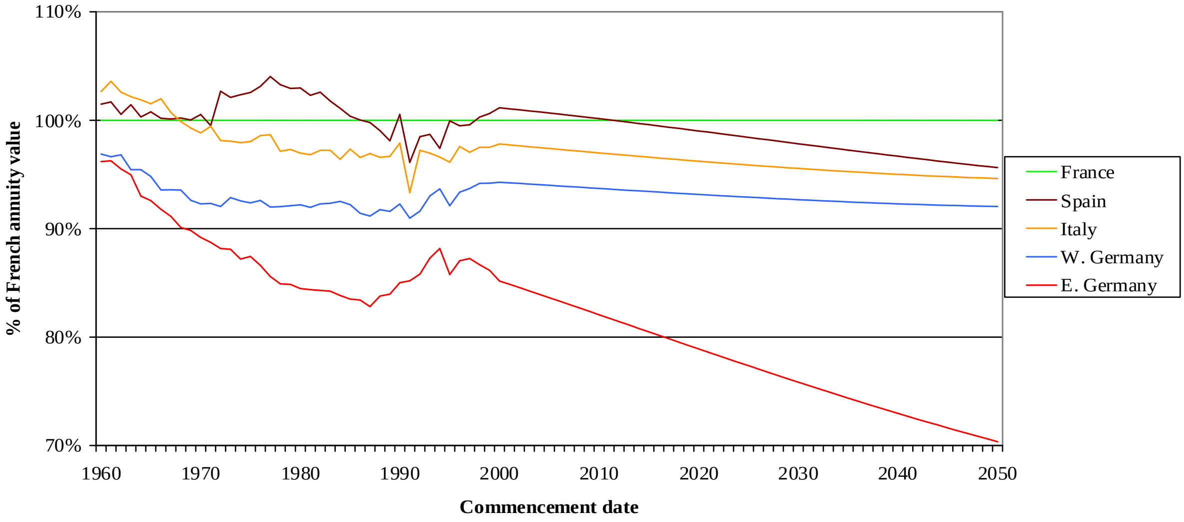
Graph 6
Weibull Model: Male annuity value comparisons for age 55 from different commencement years



Graph 7
Weibull Model: Male annuity value comparisons for age 65 from different commencement years



Graph 8
Weibull Model: Male annuity value comparisons for age 75 from different commencement years



Method 1 – Logistic model – female lives

Graph 9 sets out the capital value of annuities according to method 1 for all countries and former countries considered for females aged 65 with commencement dates from 1960 to 2050. Table 8 provides a summary of the capital value of the annuities for selected commencement dates.

Graph 9
Logistic Model : Female annuity values for age 65 from different commencement years

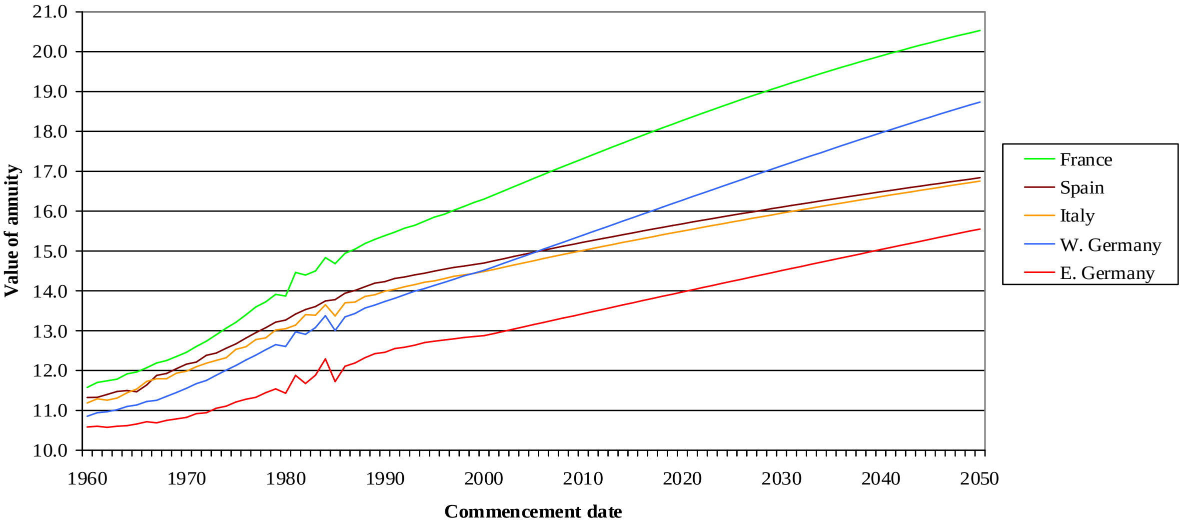
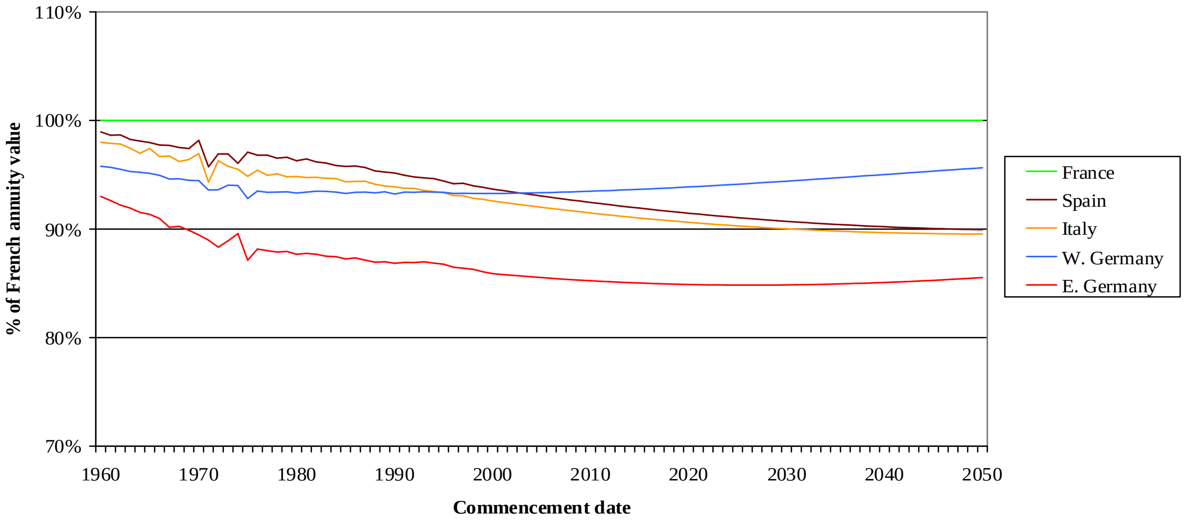


Table 8 - Value of annuities for female lives aged 65 in different countries and former countries for selected commencement dates – Logistic Model

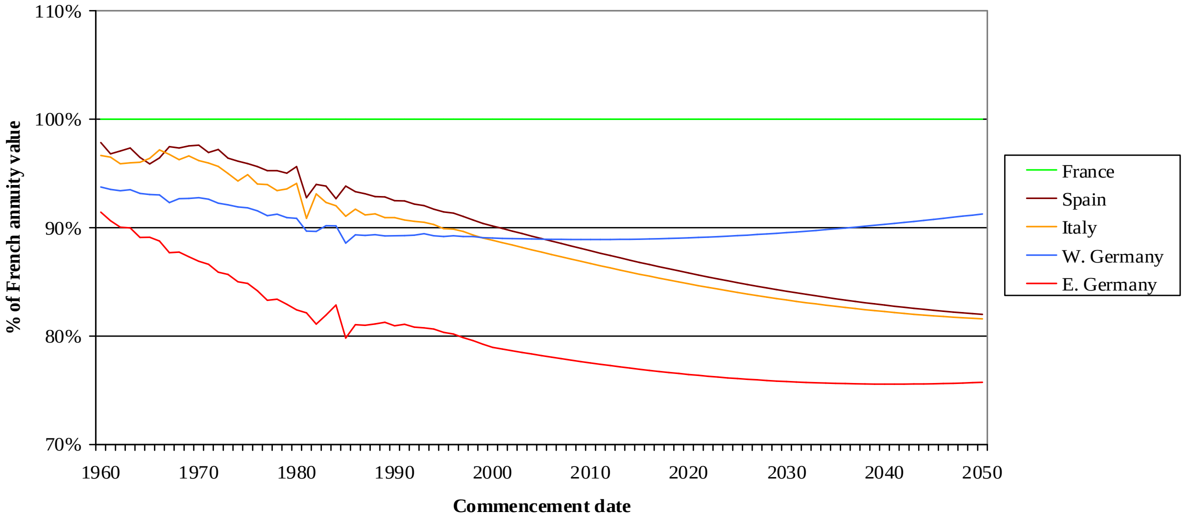
	1960	1980	2005	2020	2035	2050
France	11.57	13.87	16.82	18.27	19.53	20.53
Italy	11.19	13.05	14.76	15.50	16.16	16.75
Spain	11.32	13.27	14.97	15.68	16.30	16.84
W. Germany	10.85	12.60	14.96	16.27	17.56	18.74
E. Germany	10.58	11.43	13.15	13.97	14.77	15.55

We set out comparisons of the capital value of annuities for different countries and former countries in graphs 10, 11, and 12 relating to valuation ages of 55, 65 and 75 respectively. In each graph the comparison is with respect to the capital value of annuities for French females for each commencement year.

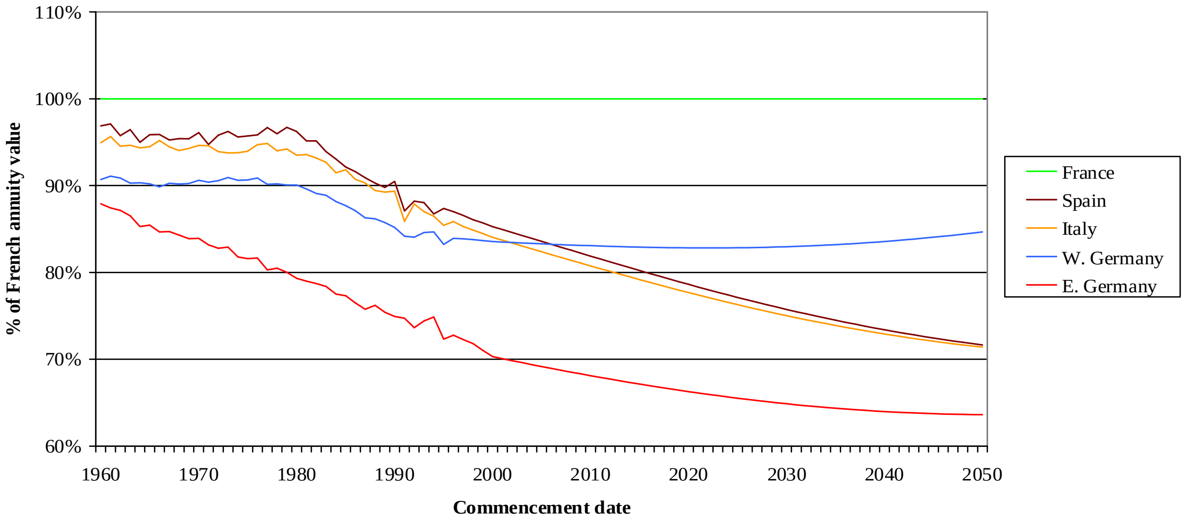
Graph 10
Logistic Model: Female annuity value comparisons for age 55 from different commencement years



Graph 11
Logistic Model: Female annuity value comparisons for age 65 from different commencement years



Graph 12
Logistic Model: Female annuity value comparisons for age 75 from different commencement years



Method 2 – Weibull distribution model – female lives

Graph 13 sets out the capital value of annuities according to method 2 for all countries and former countries considered for females aged 65 with commencement dates from 1960 to 2050. Table 9 provides a summary of the capital value of the annuities for selected commencement dates.

Graph 13
Weibull Model : Female annuity values for age 65 from different commencement years

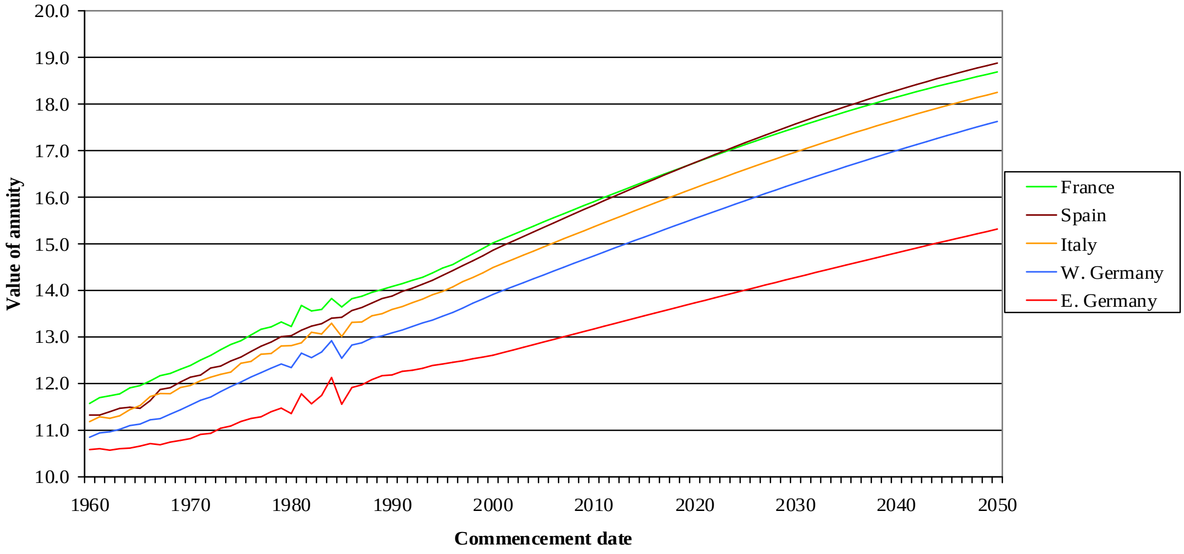
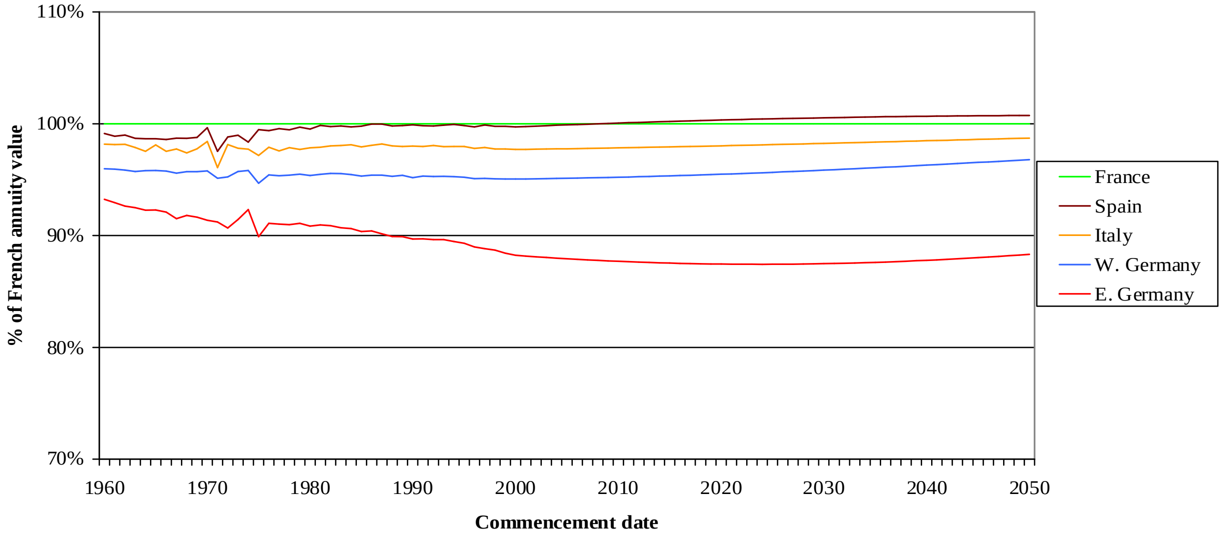


Table 9 - Value of annuities for female lives aged 65 in different countries and former countries for selected commencement dates – Weibull Model

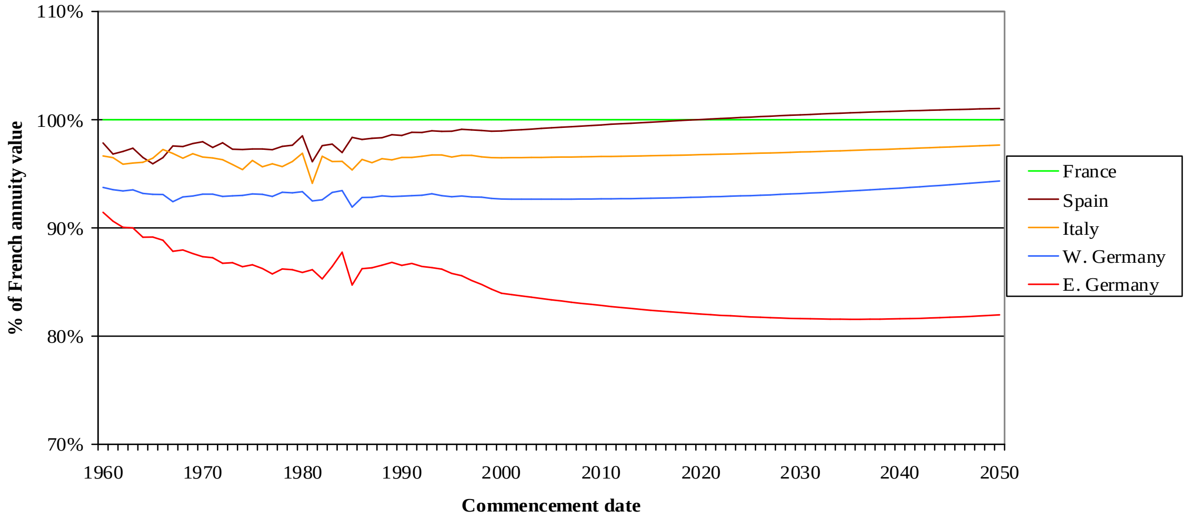
	1960	1980	2005	2020	2035	2050
France	11.57	13.23	15.47	16.74	17.83	18.69
Italy	11.19	12.81	14.93	16.20	17.32	18.25
Spain	11.32	13.03	15.35	16.74	17.94	18.88
W. Germany	10.85	12.34	14.33	15.54	16.66	17.62
E. Germany	10.58	11.36	12.89	13.73	14.54	15.32

We set out comparisons of the capital value of annuities for different countries and former countries in graphs 14, 15, and 16 relating to valuation ages of 55, 65 and 75 respectively. In each graph the comparison is with respect to the capital value of annuities for French females for each commencement year.

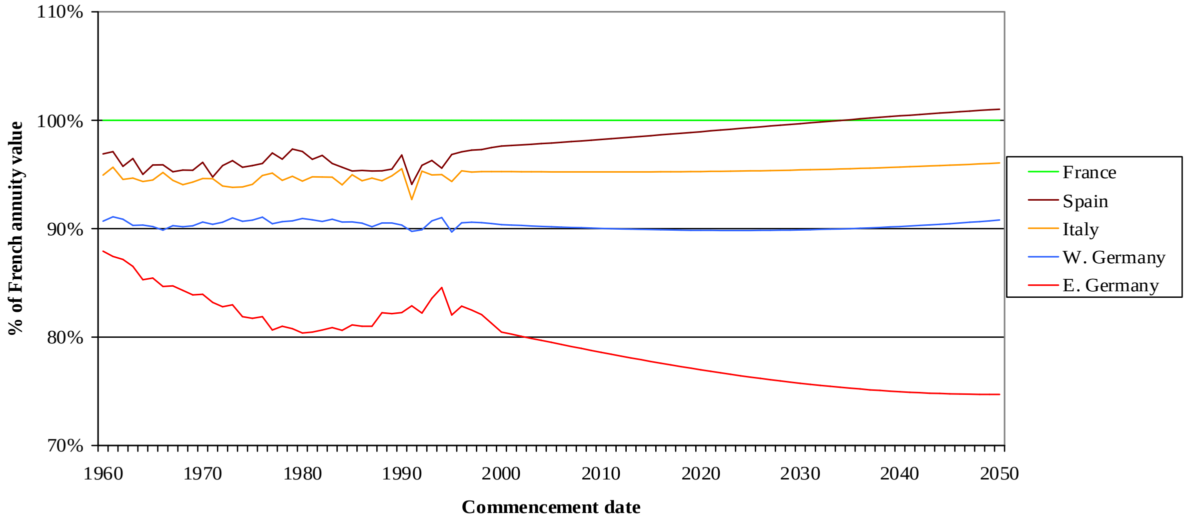
Graph 14
Weibull Model: Female annuity value comparisons for age 55 from different commencement years



Graph 15
Weibull Model: Female annuity value comparisons for age 65 from different commencement years



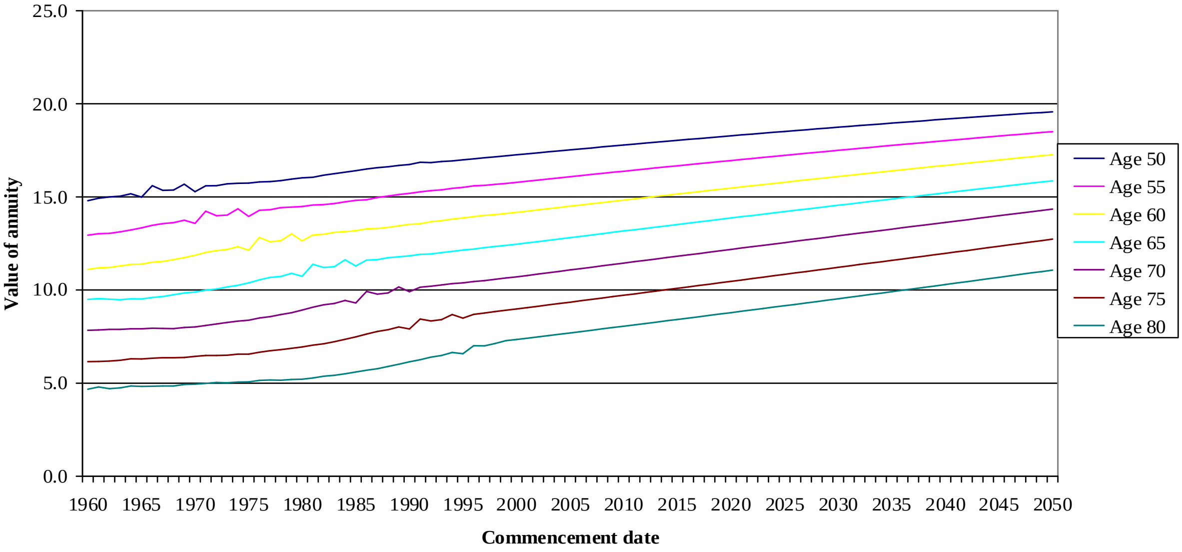
Graph 16
Weibull Model: Female annuity value comparisons for age 65 from different commencement years



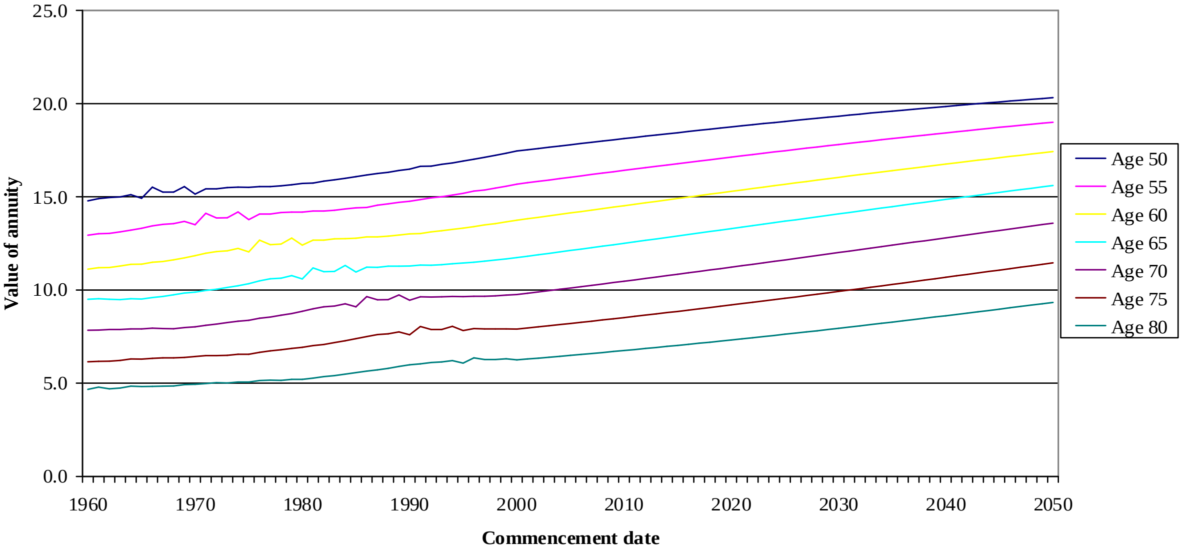
Comparison of capital value of annuities for different ages for a single country

We selected France to illustrate the effect of commencement date on the capital value of annuities at different ages. We illustrated results according to method 1 by graphs 17 and 19 for male and female lives, and results according to method 2 by graphs 18 and 20.

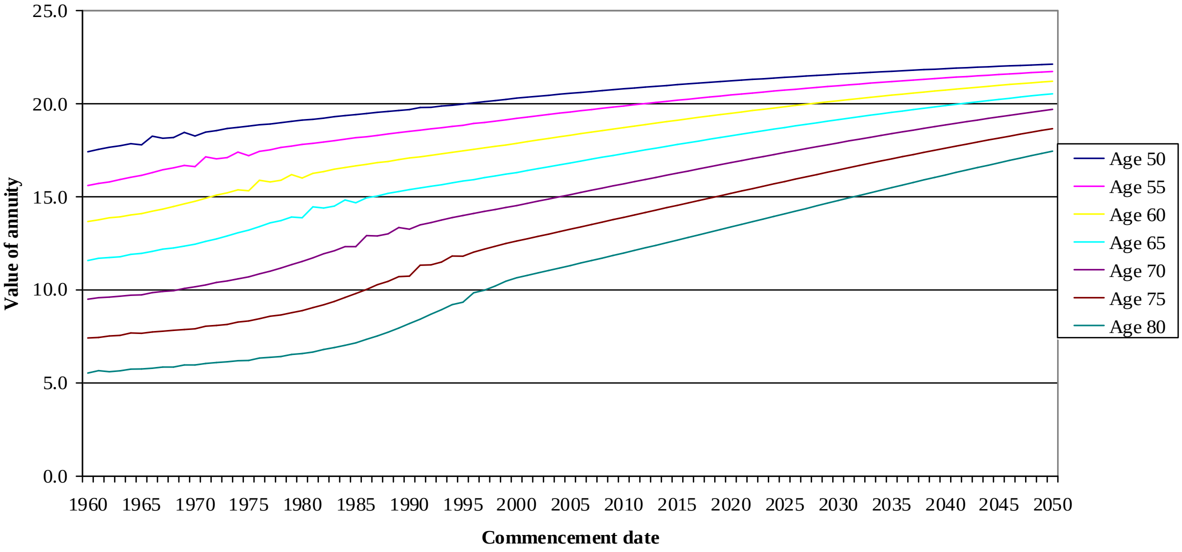
Graph 17
Logistic Model: French male annuity values from different commencement dates



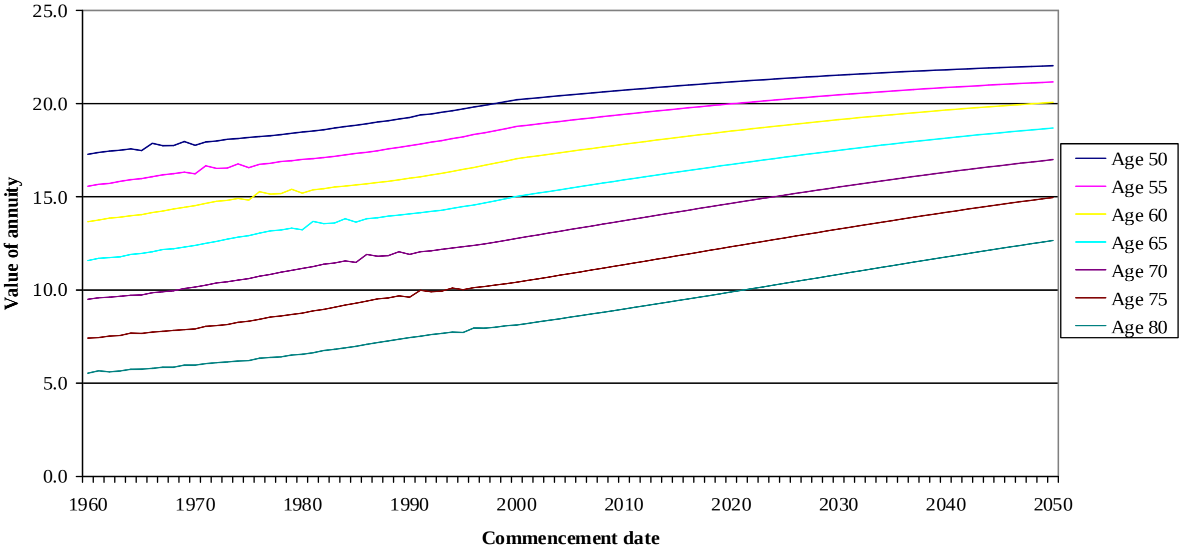
Graph 18
Weibull Model: French male annuity values from different commencement dates



Graph 19
Logistic Model: French female annuity values from different commencement dates



Graph 20
Weibull Model: French female annuity values from different commencement dates



Alternative scenario for mortality improvements

The results that we presented in respect of methods 1 and 2 could be regarded as base scenarios. The above results are based on the projection of past trends. As such the authors consider that they are a significant improvement on projections which use subjective assumptions or do not allow for the ageing of improvements. In the authors' view these methodologies should be given serious consideration by actuaries working with the pricing or reserving of annuities and pensions.

We considered alternative scenarios for each of methods 1 and 2 as to future mortality improvements. The alternative projection scenarios for each of methods 1 and 2 assume the same mortality improvements as under the base scenarios up to and including 2005, but no further mortality improvements thereafter. These scenarios are intended to be illustrative of the impact of such an assumption, rather than reflecting a probable scenario.

Comparative results in respect of method 1 are set out in table 10 for male lives and table 11 for female lives, and in respect of method 2 are set out in table 12 for male lives and table 13 for female lives.

Table 10 - Value of annuities for male lives at selected ages in year 2005 according to base and alternative scenarios for method 1

Age	**55**		**65**		**75**	
	Base	**Alt**	**Base**	**Alt**	**Base**	**Alt**
France	16.1	15.1	12.8	12.0	9.3	8.8
Italy	15.3	14.7	11.6	11.2	7.8	7.6
Spain	16.0	15.1	12.6	11.8	8.9	8.4
W. Germany	14.9	14.4	11.3	10.9	7.7	7.4
E. Germany	14.0	13.6	10.5	10.2	7.1	6.9

Table 11 - Value of annuities for female lives at selected ages in year 2005 according to base and alternative scenarios for method 1

Age	**55**		**65**		**75**	
	Base	**Alt**	**Base**	**Alt**	**Base**	**Alt**
France	19.6	17.9	16.8	15.1	13.3	11.8
Italy	18.0	17.1	14.8	14.0	10.9	10.4
Spain	18.2	17.3	15.0	14.2	11.1	10.5
W. Germany	18.3	16.9	15.0	13.7	11.0	10.1
E. Germany	16.7	15.9	13.2	12.5	9.2	8.7

Table 12 - Value of annuities for male lives at selected ages in year 2005 according to base and alternative scenarios for method 2

Age	**55**		**65**		**75**	
	Base	**Alt**	**Base**	**Alt**	**Base**	**Alt**
France	16.1	15.0	12.1	11.4	8.2	7.9
Italy	15.9	15.0	11.9	11.3	8.0	7.7
Spain	16.0	15.2	12.2	11.6	8.3	7.9
W. Germany	15.5	14.7	11.6	11.0	7.7	7.4
E. Germany	13.8	13.5	10.2	9.9	6.9	6.7

Table 13 - Value of annuities for female lives at selected ages in year 2005 according to base and alternative scenarios for method 2

Age	55		65		75	
	Base	Alt	Base	Alt	Base	Alt
France	19.1	17.7	15.5	14.3	10.9	10.2
Italy	18.7	17.3	14.9	13.8	10.4	9.7
Spain	19.1	17.6	15.4	14.1	10.7	9.9
W. Germany	18.2	16.9	14.3	13.3	9.8	9.2
E. Germany	16.8	16.0	12.9	12.3	8.7	8.3

Comparison of life expectancy from age 65 under methods 1 and 2 with EUROSTAT

We further calculated life expectancies from age 65 in a given calendar year for the different countries and former countries considered, and compared equivalent life expectancies produced by EUROSTAT. A comparison of these life expectancies from age 65 is set out in Table 14.

Table 14 - Life expectancy from age 65 according to EUROSTAT and methods 1 and 2

	EURO STAT 2000	Mtd 1 2000	Mtd 2 2000	Mtd 1 2020	Mtd 2 2020	Mtd 1 2050	Mtd 2 2050
Males							
France	16.7	17.2	15.9	20.2	18.6	24.8	23.4
Italy	16.5	15.6	15.8	17.2	18.2	19.6	22.3
Spain	16.5	16.9	16.2	19.4	18.6	23.6	22.5
W. Germany	15.7	15.1	15.1	16.7	17.5	18.9	21.6
E. Germany		14.0	13.5	15.2	14.6	17.3	16.2
Females							
France	21.2	23.9	21.4	28.8	25.1	35.7	30.9
Italy	20.4	21.4	20.4	24.0	24.0	27.5	29.6
Spain	20.4	21.8	20.9	24.4	24.8	27.8	30.9
W. Germany	19.4	20.6	19.5	24.4	22.7	30.7	28.0
E. Germany		18.0	17.5	20.3	19.7	24.0	23.2

DISCUSSION OF RESULTS

Implied improvements in life expectancy

Both mortality models that we described and used in this paper are based on the assumption that there is a time dependency to the values of the parameters underlying those models. The results given in the previous sections would suggest that this assumption has some validity over the period of the investigation 1960-1999.

For the purposes of this paper, we have fitted models of mortality separately to mortality experience in former West Germany and former East Germany reflecting the availability of data, and used these to project future mortality. An alternative approach would be to aggregate the mortality experience of the two former countries and produce a combined mortality model.

However, historical relationships in any of the countries or former countries considered may or may not be repeated in the future. Indeed, such historical relationships are likely to be the result of interaction between different factors whose relative contribution may have changed over the period of the investigation: for example, changes in the prevalence of smoking and other behavioural or environmental risk factors or the introduction and expansion of new treatments for cardiovascular disease.

With this proviso stated, both mortality models project continuing reductions in mortality rates at all ages and for both sexes. However, any good fitting extrapolative model based on recent past experience is likely to produce future mortality rates that continue to decrease.

Table 14 illustrates that projections of life expectancy from age 65 according to methods 1 and 2 generally show greater improvements for females than males. The improvements in male life expectancy from age 65 expressed as an annual rate over the period 2000-2050 vary between 0.07-0.15 years per annum for males under method 1 and 0.06-0.15 years per annum under method 2 for the different countries. The equivalent improvements for female lives from age 65 are 0.12-0.24 years per annum under method 1 and 0.11-0.19 years per annum under method 2.

Commentary on expert opinion as to the existence of limits to improvements in life expectancy

The concept of absolute limits to life expectancy was suggested by August Weissman in the nineteenth century as a mechanism by which the resources of a population would be diverted from supporting infirm, elderly members. The concept has long been discredited, in part through observations of the much longer lifespans of species in captivity and through an absence of any evidence of a biological "self-destruct" mechanism.

Christensen and Vaupel (1996) examined trends in age at death for different percentiles in the population, and observed that there does not appear to be any evidence of convergence between the different percentiles that might provide evidence of a practical limit to life expectancy which was impacting current experience.

Vaupel and Oeppen in their paper "Broken Limits to Life Expectancy" (2002) noted that improvements in life expectancy had been greater in the second half of the twentieth century than had been expected by the World Health Organisation and various government actuarial departments. They observed that examination of life expectancies over a period of 160 years in various industrialised countries showed that there was a linear relationship over that period between calendar year and the highest life expectancy in any given calendar year amongst those countries. The nature of this relationship was that the highest life expectancy increased by 0.25 years with each calendar year.

The highest life expectancy among the industrialised countries is likely to be dependent on various factors including the interplay between different countries, advances in medical and surgical treatments, changes in behavioural risk factors and the external environment. Indeed, the relative and absolute importance of each of these contributing factors is likely to have changed significantly over the course of the last 160 years.

It is therefore highly probable that such a relationship is just a coincidence. Further, the fact that this relationship has continued for such a long period of time provides no guarantee that such a relationship would continue into the future. However, in view of the uncertainty surrounding future mortality, the continuation of such a relationship is a scenario that, in the authors' view, should be considered by actuaries amongst other scenarios.

In contrast at the cellular level there is clear evidence that there are absolute limits to the number of divisions that a cell will undergo, as noted by Leonard Hayflick in his observations of fibroblasts dividing in the process of wound healing. Each mitotic division is associated with a reduction in the length of a region on the chromosome known as the telomere. It is thought that reductions in the telomere length affect gene expression and ultimately lead to cell senescence, when further divisions are impossible. It would also appear that further divisions are associated with a reduction in the capacity of the cell to function and repair itself.

At the level of the organism such changes in function and repair capacity are associated with the process of aging. Evolutionary theories of aging have been suggested that focus on the role of genes in ensuring survival to sexual maturity, either through genetic mutations that affect later life not being selected against (mutation accumulation) or through genetic mutations being selected that provide beneficial effects prior to sexual maturity at the expense of deleterious effects in later life (antagonistic pleiotropy).

Hayflick and others have suggested an alternative theory whereby genes play no direct part in the process of aging. Instead, natural selection favours genes that provide a functional overcapacity and thereby ensures survival to sexual maturity. Aging under this theory represents the result of random damage to cells that gradually reduces capacity until death occurs.

This theory leads to a concept put forward by Olshanky, Carnes, and Cassel (1990) of a "biological warranty" or a practical upper limit to life expectancy. This is, in effect, the result of the combination of a given level of functional overcapacity and a given rate of aging. Proponents of the "biological warranty" concept note that the practical upper limit may be close and that extrapolation of past trends in life expectancy is not supported by the underlying biological processes. They further note that, with the possible exception of caloric restriction, there do not appear to be at present any strategies for slowing, let alone reversing, the pace of the aging process.

However, the current consensus amongst demographers and biogerontologists is that there is no reason to suppose that such treatments could not be developed in the future. Such treatments might include the use of the enzyme telomerase which restores the length of telomeres and appears to rejuvenate cell function as evidenced by initial in vitro experiments. The impact of any such treatments on life expectancy would depend on the degree of penetration in the population, the monetary costs associated with treatment and the possibility of adverse side effects.

We noted previously that changes in life expectancy are also a function of changes in behaviour risk factors. Reductions in the prevalence of smoking in the second half of the twentieth century in some countries have had a significant effect on lung cancer and cardiovascular disease. Changes in diet, in patterns of exercise and in the external environment may have an adverse impact on health and life expectancy. For some countries it is possible that these adverse effects may lead to reductions in life expectancy despite advances in medical intervention.

CONCLUSIONS

We would draw the following conclusions from our investigations as presented in this paper:

- Tables 1-4 illustrate that logistic and Weibull distribution models can provide a broadly similar level of fit to actual mortality experience at older ages in any given calendar year, but that the Weibull distribution models show a clearer pattern of time-dependent parameters over the investigation period.
- Table 5 indicates that the use of a time-dependent Weibull distribution models provides a generally closer fit to underlying mortality experience than using a time-dependent logistic model for either sex
- The use of past trends to project future mortality according to either of the methods described leads to the result that future mortality improvements are expected to be higher when past mortality improvements have been higher. Possible arguments can be put forward for and against such a relationship occurring in practice.
- Both methods project increasing values for annuities with later commencement dates for all ages, sexes and countries considered. However, these results should be considered in the light of current understanding as to possible practical limits to life expectancy.

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