THE USE OF MONTE CARLO SIMULATION OF MORTALITY EXPERIENCE IN ESTIMATING THE COST OF PROFIT SHARING ARRANGEMENTS FOR GROUP LIFE POLICIES.

By LJ Rossouw and P Temple

ABSTRACT

This paper expands previous work that has been done on a methodology for simulating the mortality experience of employer based group benefit policies using Monte Carlo simulation with specific focus on profit sharing arrangements. We show that results of these simulations could be simplified to a formula that may be applied in practical situations. We investigate the impact of varying assumptions and we also suggest other applications for this methodology.

KEYWORDS

Monte Carlo simulation; profit shares; profit commission; reinsurance.

CONTACT DETAILS

Rossouw, Louis, Gen Re, PO Box 444, Cape Town, 8000, South Africa. Tel: +27 21 412 7712. Fax: +27 21 412 7782. E-mail: lrossouw@genre.com

Temple, Peter, Gen Re, PO Box 444, Cape Town, 8000, South Africa. Tel: +27 21 412 7706. Fax: +27 21 412 7782. E-mail: ptemple@genre.com

1 INTRODUCTION

In South Africa, profit shares have been available on group business for many years. In many cases profit shares were added at the end of a quotation process after all the "real" pricing work had been completed. This may have been as an afterthought to enhance the offering for the client or may have been at the request of the client. They were often added with little or no change to the rates. Where a loading to the rates was made it would usually have been done using fairly approximate and ad hoc methods. The justification may have been that the rates were conservative therefore a profit share could be added for little or no cost.

As the Group market has become evermore competitive, pricing for the underlying group life benefit has reduced (sometimes to the extent that profits are difficult to achieve). In this competitive market profit shares are still requested but the implicit margins of the past are not there to cross-subsidise the cost. It has become important to price profit shares correctly as they can lead to the insurer, at best, struggling to reach its profit margin.

One of the tools that is useful for pricing profit shares is Monte Carlo simulation. In this paper we developed a model of the total death claims of a group using Monte Carlo simulation. We then used this model to analyse profit share costs.

We acknowledge that there are other alternative methods for pricing for profit shares. We have elected to focus this paper only on this method.

2 METHODOLOGY

The underlying model is based on a group benefits policy insured with an insurer for one year. The benefit assumed throughout is a lump sum benefit and for the purposes of this discussion we assume it is a death benefit. The methodology would be relevant to other lump sum benefits as well. Typically the group would be an employer-based group with the owner of the policy being the employer or a pension fund. The policy would be used to provide death-in-service benefits to the employees of the company.

For the analysis done, the lives were assumed to be independent. The validity of this assumption may be problematic in cases where the employees share a common location or environment and/or other risk factor. For example, a group of doctors working in separate locations could be exposed to the same infectious disease.

The general methodology employed was to simulate a group's experience using Monte Carlo simulation and calculate the margin for a given level of profit share such that the profit target of the insurer is met.

Work on this type of model has been done in the past, but we believe that we have broadened the analysis to measure the impact of varying some of the underlying assumptions. We also wanted to show that a practical solution could be derived to offer a relatively quick and easy calculation of the cost.

2.1 Monte Carlo Simulation

To simulate the claims cost related to the *i*th individual in our insured group we need to know the probability of death q_i, as well as the sum assured for that life, S_i.

The claim for the *i*th individual in a year would be

$X_i \cdot S_i$

$X_i = 0$ where $Z_i \leq q_i$ and

$X_i = 1$ where $Z_i > q_i$

with Z_i distributed uniformly between 0 and 1.

It can be seen that X_i is a Bernoulli distribution.

The total claims for the insured group in a year would be

$$C = \sum_{i=1}^{N} X_i \cdot S_i$$

where N is the number of lives in the group benefit arrangement. If q_i is constant and $S_i = 1$ for all *i* then it can be seen that C follows the binomial distribution.

We can simulate X_i by generating a pseudo-random number and comparing it to q_i. Thus we sum the benefit assured for all lives where the pseudo-random number is less than or equal to q_i. This gives us a single iteration of our simulation. Repeating this process many times gives us the distribution of C.

It is possible, and quite practical, to do this simulation based on the actual member data using individual mortality rates and sums assured.

For the purposes of this analysis we simplified the process by simulating the number of claims separately from the claims amount. That is we simulated $\sum_{i=1}^{N} X_i$ assuming a constant q_i. We then assumed S_i are distributed exponentially independent of q_i. We then used the inverse of the cumulative distribution function of the exponential distribution to generate a random sum assured given a pseudo-random number distributed uniformly between 0 and 1. Both Thornley (2001) and Czernicki, Harewood & That (2003) provide good descriptions of this kind of process.

2.2 The Underlying Group Policy

It is assumed that the pricing for the group policy is based on the following formula:

$$P \cdot (1 - e - l - \pi) = R$$

Where:

P is the gross premium.

e is the expense margin of the policy.

l is the margin for profit share.

π is the target profit margin of the insurer.

R is the pure risk premium for the group. This is the premium to cover the expected amount of claims and can be expressed algebraically as follows:

$$R = \sum_{i=1}^{N} q_i \cdot S_i$$

P and *R* are assumed to be payable in advance of the one-year of cover. For the purposes of this analysis discounting is ignored.

2.3 Profit Sharing Basis

It is assumed that the profit share is calculated as

$$Y = Max\left[F \cdot (P \cdot (1 - e - l - \pi) - C), 0\right]$$

where F is percentage of profit shared.

Y can also be expressed as

$$Y = Max\left[F \cdot (R - C), 0\right].$$

Generally the expense and profit margin (and possibly also the margin for profit share) would be combined for presentation purposes when quoting.

The model uses a profit share structure where the expense and profit margins in the profit sharing structure are identical to the actual margins of the insured. This may not be always the case. In this case it may be easily allowed for with explicit modelling but care is needed when using simplifying assumptions.

For the purpose of this paper it was assumed that l is the unknown. We aimed to derive a value of l such that the profit target is expected to be met for a given profit share structure. The analysis could also have been reversed to find out how much of profit could be shared (F is the unknown) given a specified margin for profit shares.

Often a profit share may have a carry over of losses from previous years. However in a yearly renewable market this would add little protection for the insurer as the group may opt to insure with another insurer to avoid having to help the insurer recover the loss. If however a insurer can be certain of retaining a group policy for a number of years, and assuming that the profit share is payable every year, one could develop an approach to simulating losses carried over. A possible solution may be to group a number of simulations together and let losses carry over from one simulation to the next in each group. In this case each simulation would represent a year.

For example, one could choose a grouping of 5 simulations. This would assume a persistency of 5 years and that the losses in earlier years could be recouped in the following years. The number of years should be chosen conservatively with reference to the likely persistency of the group policy. One could also use explicit persistency assumptions. Results from future years may be discounted. One would also need to consider potential membership changes over the period

2.4 Profit Target

The value of l is calculated such that the expected profit for the insurer, including allowances for claims and profit share, will equal the targeted profit. This can be expressed as

$E[P - e \cdot P - C - Y] = \pi \cdot P$ given that actual expenses are as per our assumption of $e \cdot P$

which becomes

$$P - e \cdot P - E[C + Y] = \pi \cdot P$$

or

$$P \cdot (1 - e) - R - E[Y] = \pi \cdot P.$$

Using

$$P \cdot (1 - e - l - \pi) = R$$

we can derive

$$l = \frac{E[Y]\cdot(1-e-\pi)}{R+E[Y]}.$$

For the purpose of further analysis we assumed that the expense and profit targets are both 0%. This simplification was not needed to simplify modelling but merely to remove distracting elements from the analysis. We can do this as long as the premium used in the profit share calculation remains the expected risk premium. To express this algebraically, we simply set e and π to 0. Y is independent of these numbers as was shown above.

Removing the profit and expense margins, the question became "How much does the risk premium need to be loaded to allow for giving a share in profits to the group, without leading to losses for the insurer?"

3 RESULTS

3.1 Variations in parameters

3.1.1 Initial Scenario

Initial analysis was done using a group with the parameters as per Table 1. We also assumed that the sum assured was independent of mortality. It is clear from these assumptions that the risk premium (R) is 500,000.

Number of Lives (N)	5,000
Mortality (q_i)	0.002 for all lives
Sum Assured (S_i)	Distributed exponentially with an average of 50 000
Group's Share of Profit (F)	50%
Simulations	5,000

Table 1. Details of basic scenario

This scenario was similar to Method 1 of Thornley (2001). Our scenario may have been somewhat simplistic but we provide an analysis based on an actual group in APPENDIX A.

The resultant margin (l) was estimated at 8.1%.

Figure 1 contains a plot of the frequency of different levels of simulated claims.

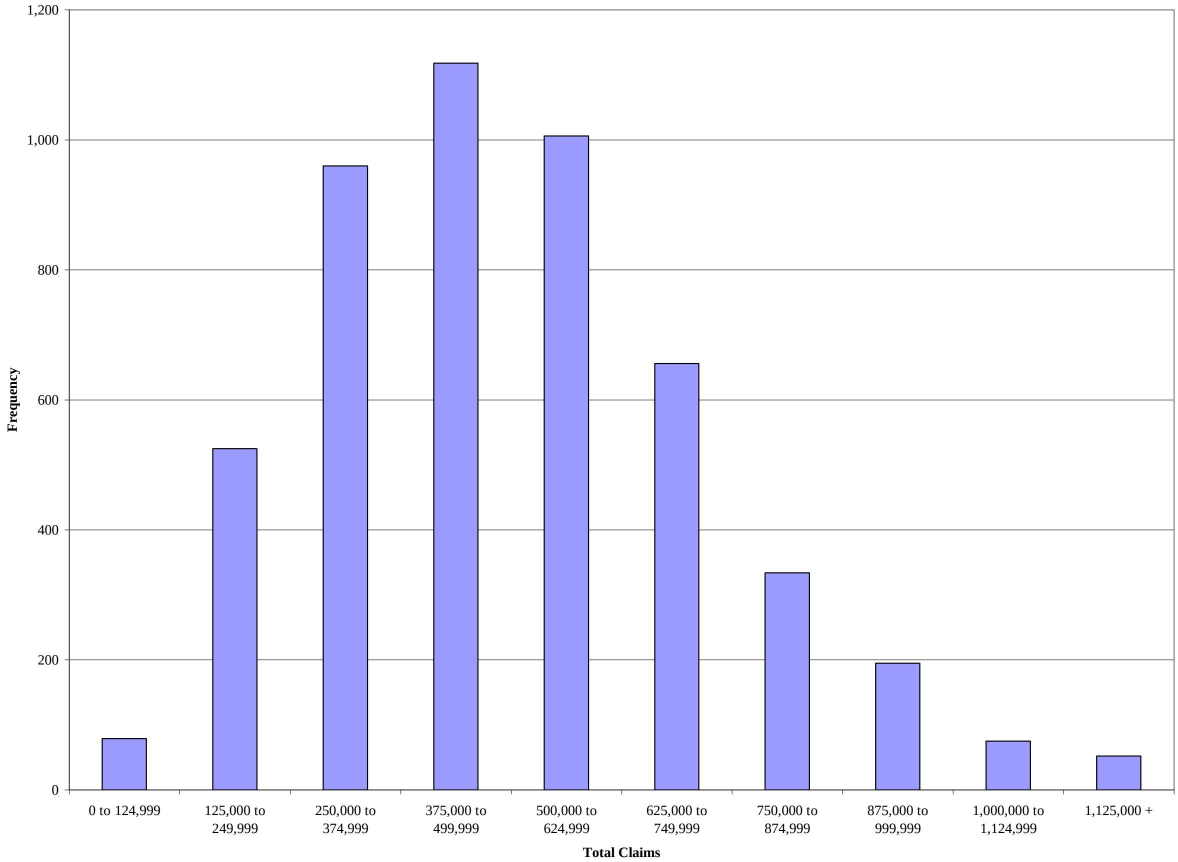

Figure 1. Frequency plot of total claims

Figure 2 contains a plot of the insurer's profit with and without a profit share for each simulation. The X-axis shows the profit the insurer would have made had there been no profit share compared to the profit the insurer would have made with a profit share on the Y-axis. This profit included an allowance for the loading for the relevant profit share.

The following relationship applies:

Profit with profit share

= *Profit without profit share*

plus *Loading for Profit Share*

less *Profit Share Payment*

Thus where there is a loss the insurer is simply reducing that loss by the loading for the profit share. Where there is a profit the insurer is increasing its profit with the loading, but reducing it by paying some of the profit back to the group policyholder. The gradient of the line where a profit share is paid is $1-F$. Otherwise, it is 1.

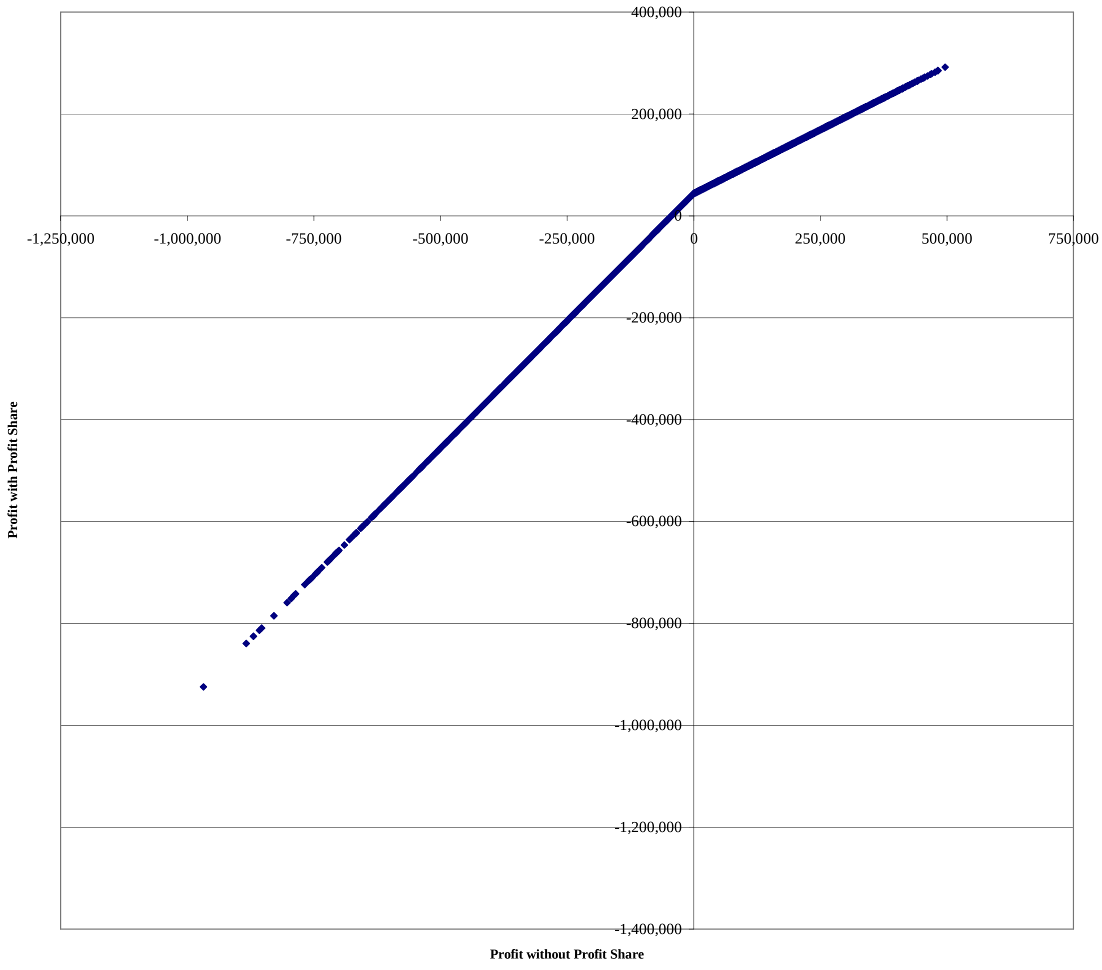

Figure 2. Plot of insurer's profit per simulation with and without a profit share.

Figure 3 plots the distribution of the insurer's profit before and after the profit share is applied. This shows the impact on the distribution of the insurers profit. Extreme losses are reduced (due to the profit share loading) and extreme profits are reduced (by the profit share applying).

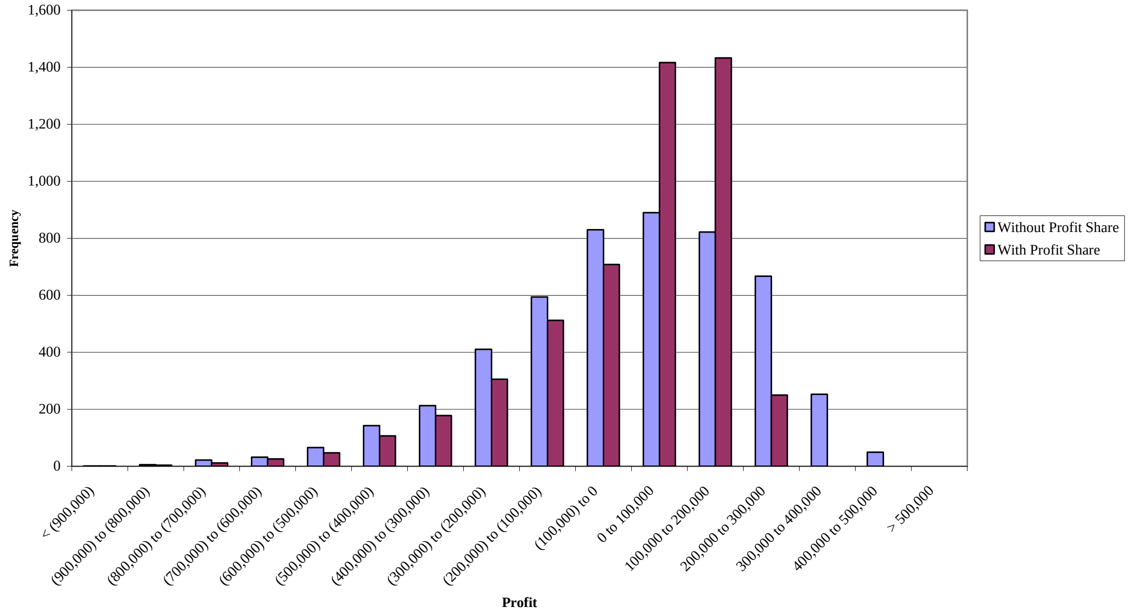

Figure 3. Plot of distribution of insurer's profit with and without a profit share.

3.1.2 Variations in number of lives

The simulation above was repeated for various numbers of lives. The corresponding values of l are plotted in Figure 4 and Figure 5 that highlights results for smaller groups. Other assumptions were as per 3.1.1. Each variation was run 1,000 times as opposed to 5,000 times to reduce the time required for the calculations. This should not have affect the results significantly as the overall shape and trends in the values were of more importance than the exact value at any specific point.

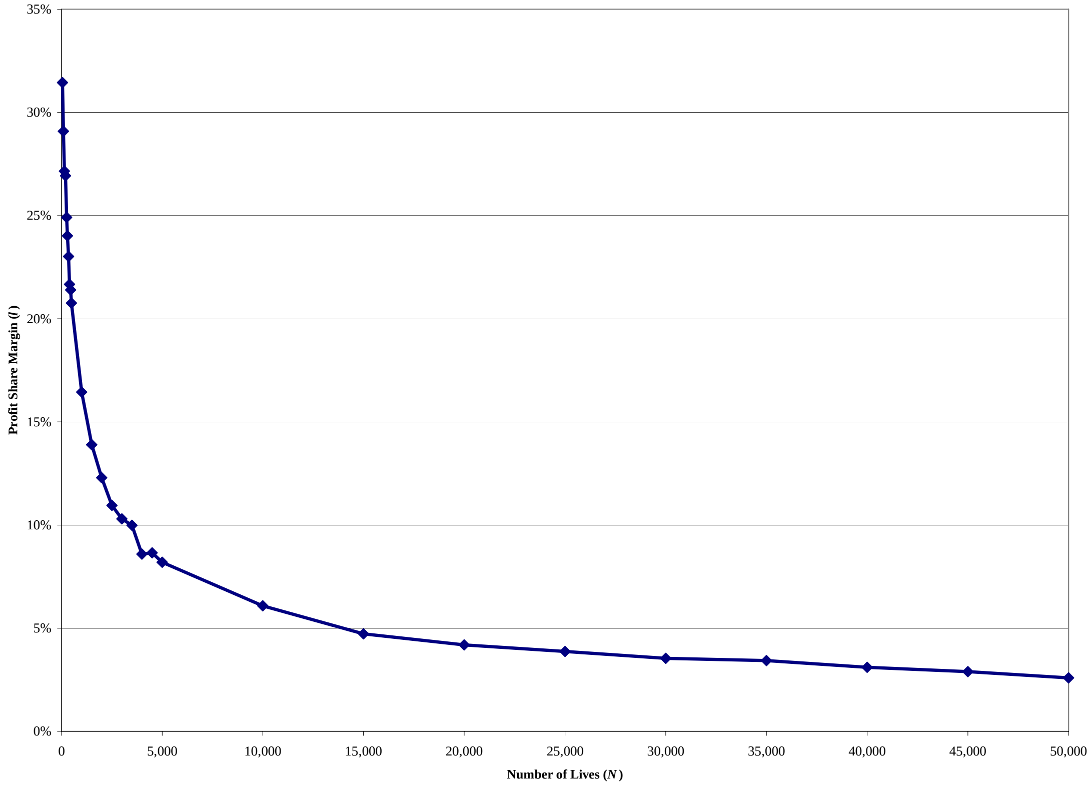

Figure 4. Plot of value of profit share margin (l) for different numbers of lives (N)

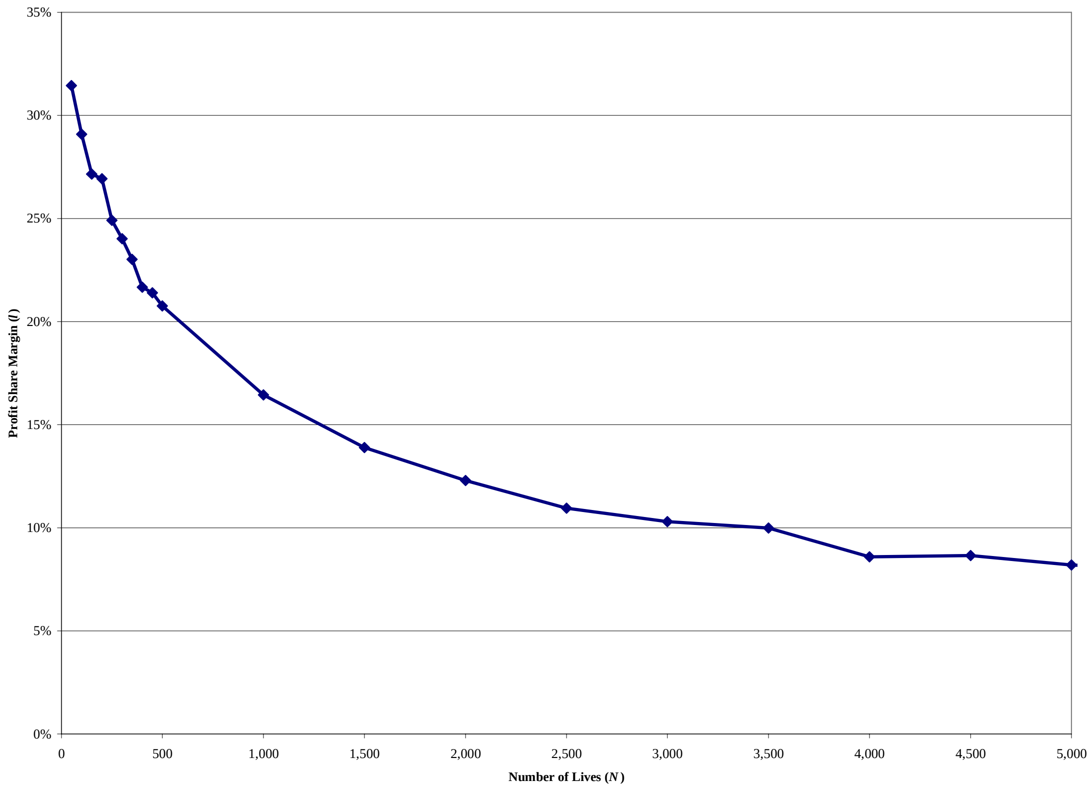

Figure 5. Plot of value of profit share margin (l) for different numbers of lives (N) – Details for lower numbers of lives

Figure 4 and Figure 5 show that the margin required for a profit share decreases as the number of lives in the group increases. This is intuitive because, as the number of lives increases, the experience would be less volatile, and less volatile experience results in a lower cost of profit shares.

3.1.3 Variations in mortality

Simulations were also repeated for various levels of mortality. Other assumptions were as per 3.1.1. Figure 6 illustrates the results. The fact that the margin decreases with increasing mortality conforms to expectations. From the variance of the binomial distribution we know that the coefficient of variance of number of deaths increases for lower mortality rates. Thus we expect claims cost to be less volatile if we expect more deaths.

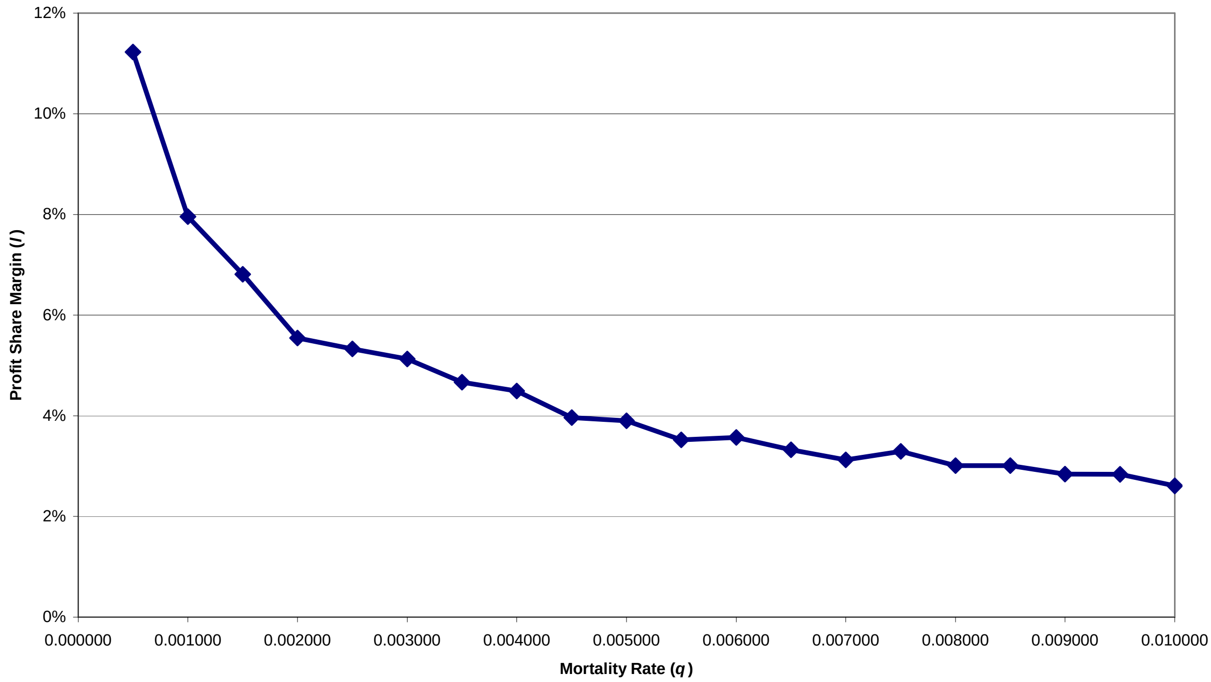

Figure 6. Plot of value of profit share margin (l) for different values of mortality (q)

3.1.4 Claim Amount Distribution

The effect of the added variability due to claim amount distribution was measured by replacing the exponential distribution with a constant value. Other assumptions were as per 3.1.1. The resultant estimate for the profit share loading was 5.9% (compared to 8.1%). Figure 7 illustrates the effect on the total claims distribution. Here the distribution assuming constant claim amounts is not as wide as when we assumed exponential claim amounts.

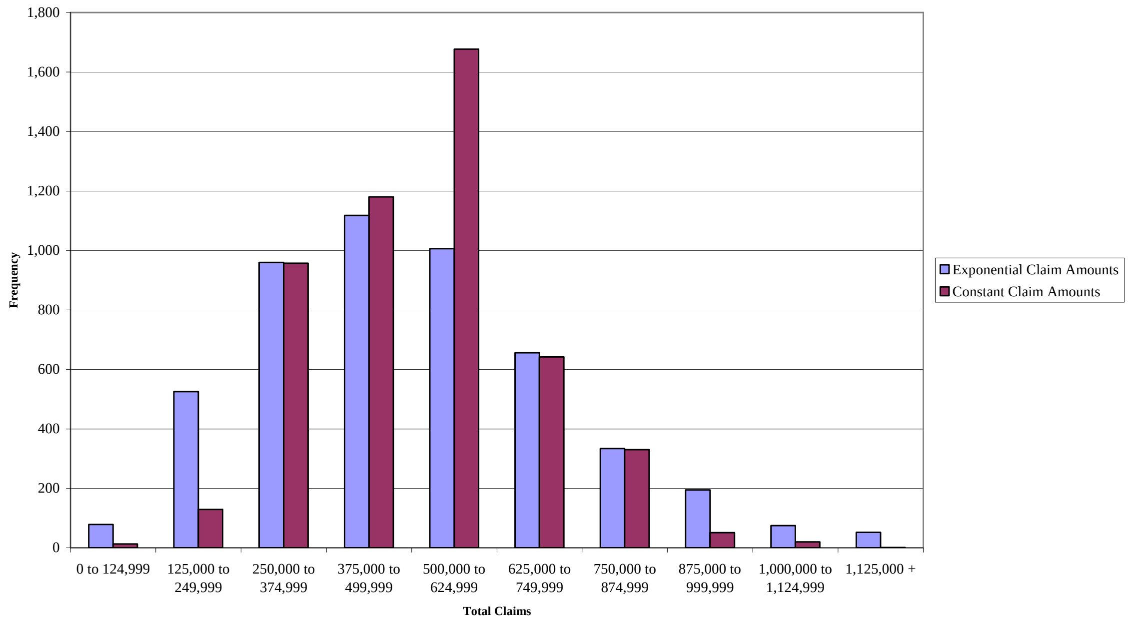

Figure 7. Distribution of total claims with constant claim amounts and with exponential claim amounts.

3.2 Approximation of Results

To approximate the results we needed a wide range of potential schemes to test our results on. We obtained this by using different combinations of number of lives and mortality level as sample schemes. The number of lives was assumed to vary from 50 to 10,000. Mortality was assumed at various values from 0.000500 to 0.009500. Other assumptions were as per 3.1.1. For each combination of mortality and number of lives (i.e. expected number of deaths) we calculated the required margin (l).

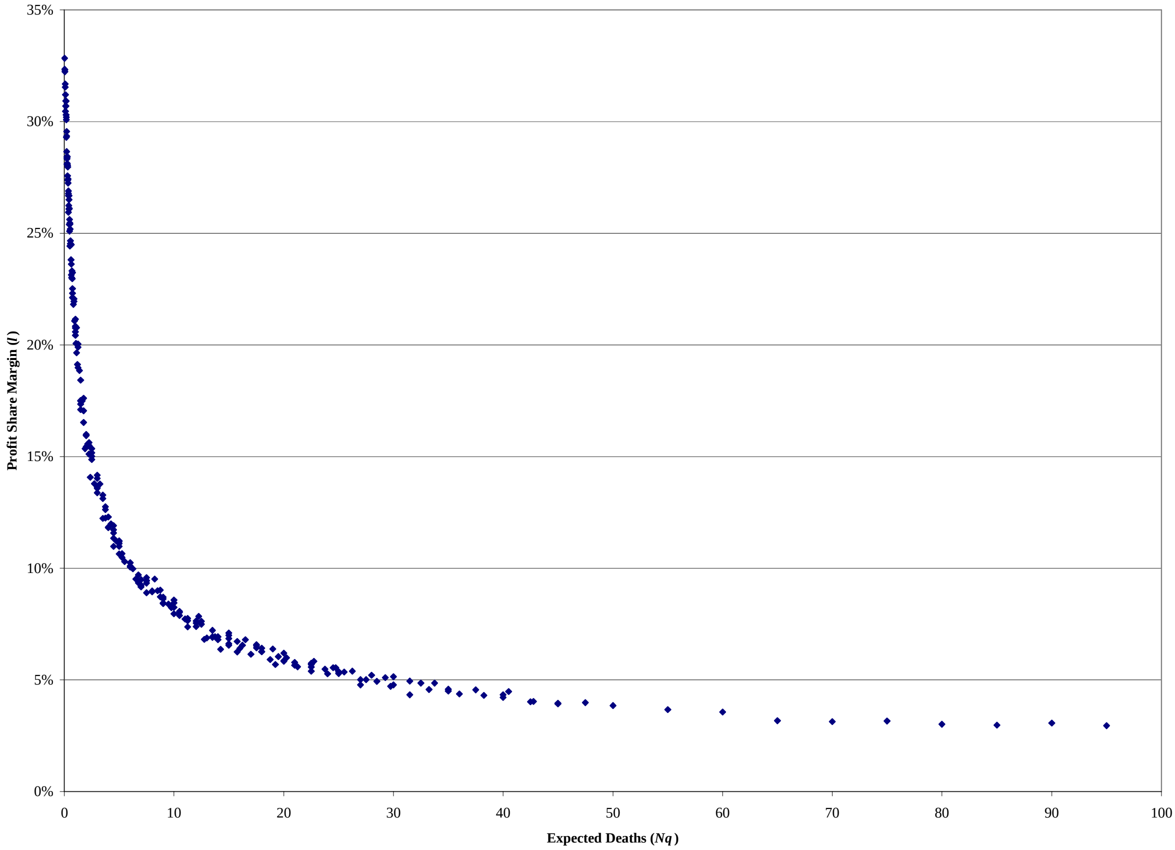

Figure 8. Plot of value of profit share margin (l) various numbers of expected deaths

Figure 9 plots the approximate coefficient of variation of the number of deaths and the profit share loading. The coefficient was calculated as the standard deviation of the number of deaths divided by the mean. This can be expressed as

$$\frac{\sqrt{Nq(1-q)}}{Nq}.$$

Assuming $(1-q) \approx 1$ this becomes

$\dfrac{1}{\sqrt{Nq}}$ or 1 divided by the square root of expected deaths.

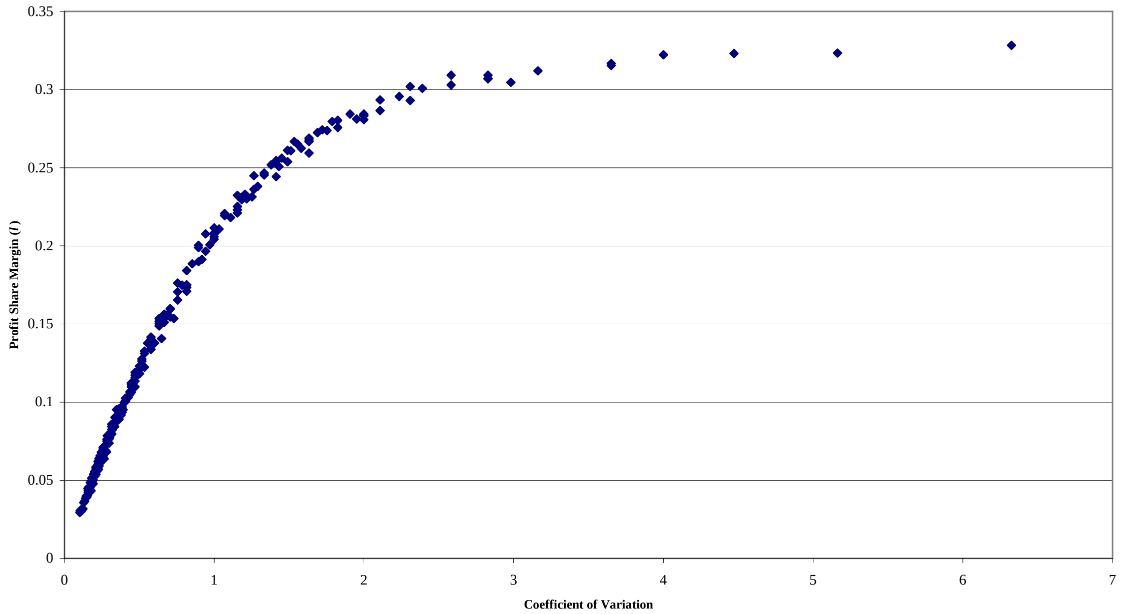

Figure 9. Plot of value of profit share margin (l) for values of coefficient of variation

In Figure 10 detail from Figure 9 is plotted (where the coefficient of variation is below 1). A line has also been fitted through the points where $\frac{1}{\sqrt{Nq}} < 1$ or where expected deaths (Nq) greater than 1.

Thus, approximations for the value of l might be made using the formula:

$$l \approx 0.2076 \cdot \frac{1}{\sqrt{Nq}} + 0.0135 \text{ where } \frac{1}{\sqrt{Nq}} < 1.$$

Note this result applies to a 50% profit share structure. Varying values of F can be incorporated in the formula using the results of 3.3.1 below. This kind of formulae would be helpful in implementing profit share pricing on a simple basis on a group quotes system.

Where expected deaths are below 1 it is very unlikely that a profit share would be offered, because the experience would be extremely volatile and this would result in a prohibitive cost of profit share.

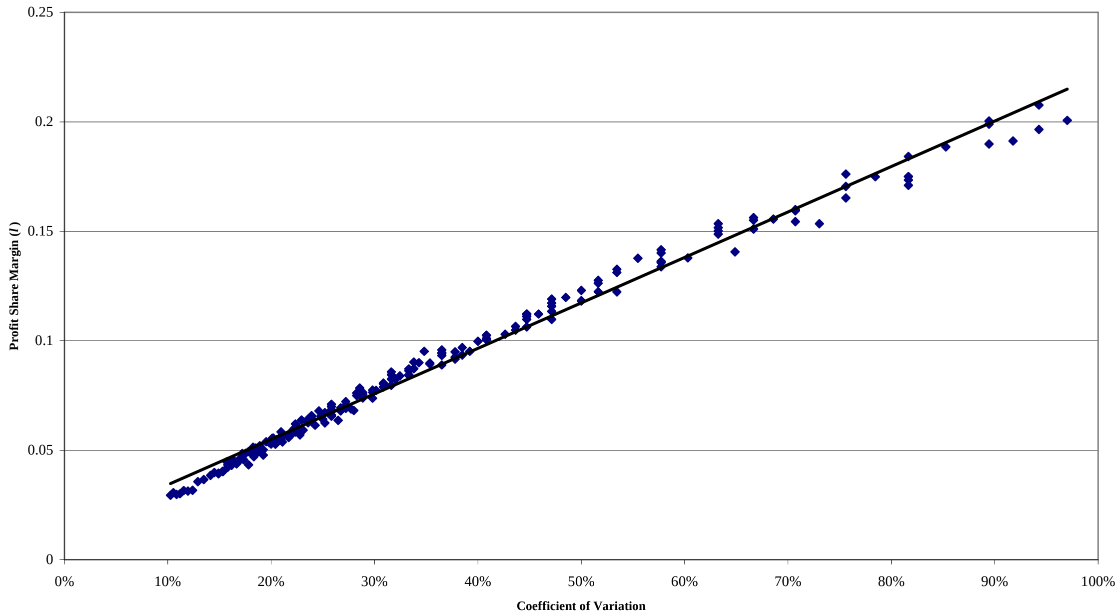

Figure 10. Plot of value of profit share margin (l) for values of coefficient of variation (detail with trend line)

3.3 Variation in percentage and calculation of profit shared

We investigated variations in the percentage profit shared (F) as well as a sliding scale profit sharing structure below.

3.3.1 Variations in percentage profit shared

The percentage profit shared (F) was varied between 0% and 100% in increments of 1%. Other assumptions were as per 3.1.1. The profit share loading was estimated in each case (using 1,000 simulations each time). Figure 11 illustrates the results.

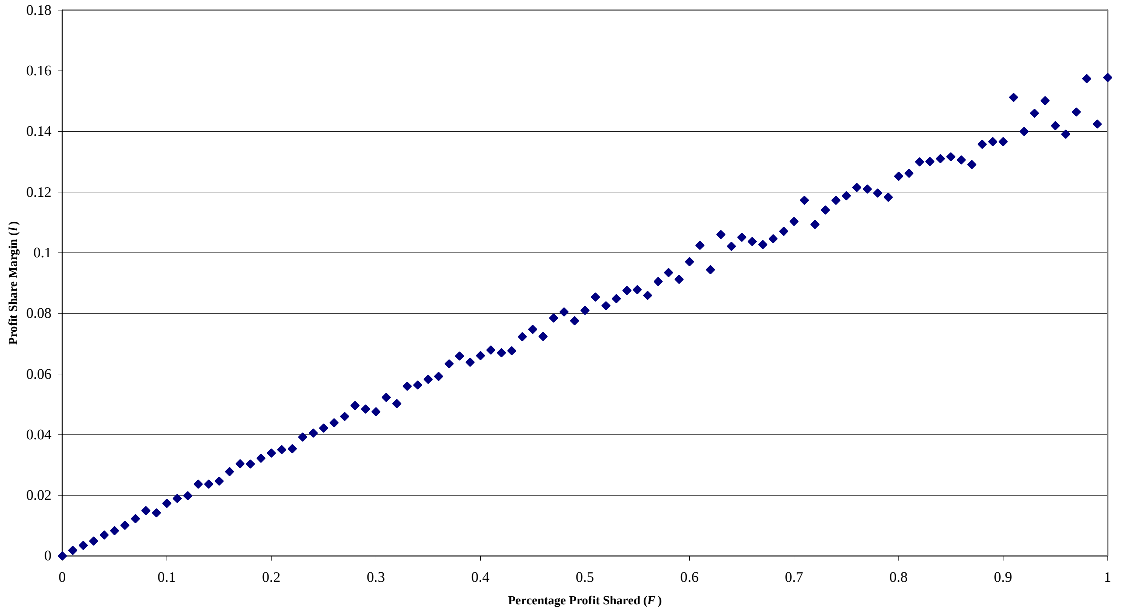

Figure 11. Plot of value of profit share margin (l) for varying percentages of profit shared (F)

Figure 11 shows a linear relationship. This follows from

$$l = \frac{E[Y] \cdot (1 - e - \pi)}{R + E[Y]}$$

which can be rewritten as

$$l = \frac{F \cdot E[X] \cdot (1 - e - \pi)}{R + F \cdot E[X]}$$

Where $X = Max[(R - C), 0]$

Given that $F \cdot E[X]$ is relatively small compared R an approximate linear relationship holds between F and l.

3.3.2 Different profit sharing structure

Profit sharing structures do not always follow a fixed percentage of profit. A sliding scale of profit sharing as a percentage of premiums may apply. Here we assumed that the premium used in the sliding scale is the risk premium (*R*). The calculation could easily be adjusted if this is not the case. Table 2 contains the sliding scale we investigated

Profit as percentage of premium	Profit share percentage
First 5%	0%
Next 25%	50%
Thereafter	100%

Table 2. Alternative Profit Sharing Structure

The resultant loading for the above structure was 8.3%. This is similar in loading to a simple structure with a profit share of 50%. However when we compare the profit before and after the profit share with that of the simple structure, the difference becomes apparent. Figure 12 charts this.

The graph is similar to Figure 2 except that after a point the insurer can make no further profits. This is because any further profits are passed back to the group.

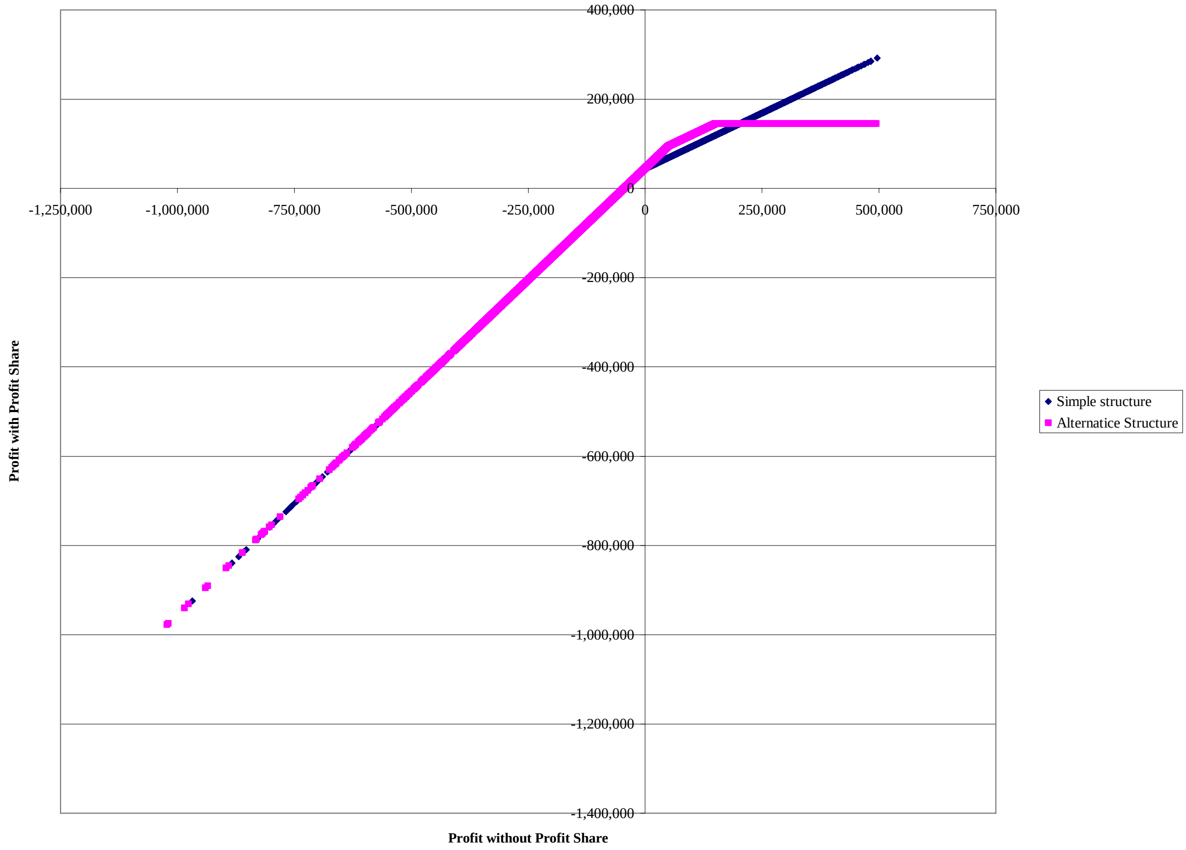

Figure 12. Plot of insurer's profit per simulation without a profit share and with a profit share under two profit sharing structures

3.4 Parameter Risk

The results above allow for volatility of claims due to random variation only. They do not allow for volatility resulting from uncertainty surrounding the assumed mortality rate. In practice we cannot be 100% certain of the mortality that would apply for a given group of lives.

Quantifying this uncertainty is difficult and would depend on the method adopted in deriving the estimate for mortality. We illustrated the potential impact of parameter uncertainty by repeating the initial calculations in 3.1.1 with an assumed distribution of the mortality of the group.

We assumed parameter risk would be as per Table 3. This was allowed for by changing the mortality rate for a quarter of the simulations to a level 25% higher, for another quarter to a level 25% lower, than the initial assumptions. For the remainder of the simulations the mortality was left unchanged from the initial assumption.

Impact on parameter	Probability
Actual mortality 25% higher than assumed	25%
Mortality assumption correct	50%
Actual mortality 25% lower than assumed	25%

Table 3. Parameter Risk Assumptions

Other assumptions were as per 3.1.1.

The resultant profit share loading (*l*) was estimated at 8.6% (as opposed to 8.1% without parameter risk). The chart below illustrates the greater variation in the total claims distribution.

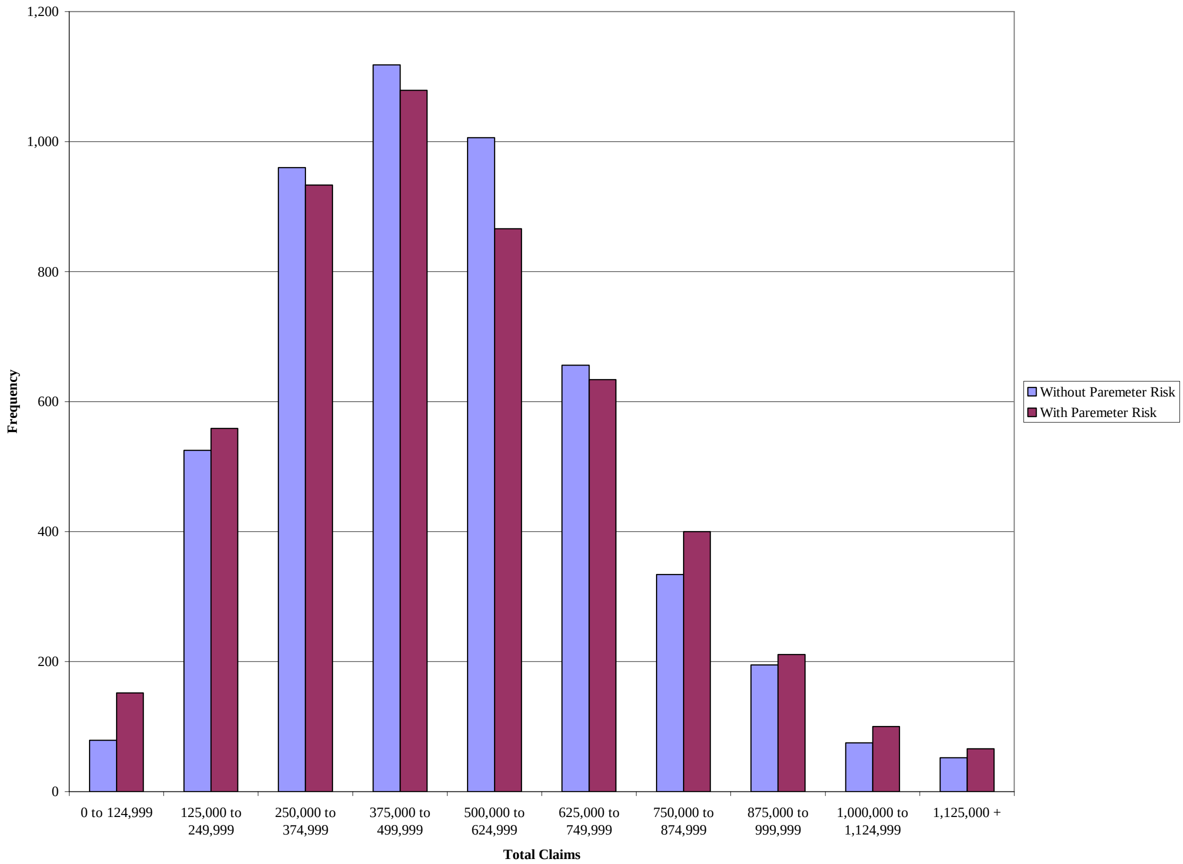

Figure 13. Frequency plot of total claims – with and without parameter risk

3.5 Heterogeneity

For all modeling until now we assumed a homogenous group of lives. We model the potential impact of heterogeneity assuming a group consisting of two categories of members.

	Category A	Category B	Combined
Number of Lives (N)	500	4,750	5,250
Mortality (q_i)	0.001000	0.002000	0.001613
Sum Assured (S_i)	Distributed exponentially with an average of 240 000	Distributed exponentially with an average of 40 000	Average of 59,048 (weighted by number of members)
Expected Number of Claims	0.5	9.5	10
Average Claim	240,000	40,000	50,000
Risk Premium	120,000	380,000	500,000

Table 4. Assumptions for category of members

We have set assumptions for this group such that the expected number of claims, the average claim and therefore the risk premium, stays the same as for the original group in 3.1.1. Note that the other group had slightly fewer members however, as the main determinant of profit share cost is the expected number of deaths, we felt that this difference would not be significant for the assessment of the impact of heterogeneity.

When we model this group, using the appropriate assumptions for each category of member, we obtained an estimated required loading of 9.7% (compared to 8.1%).

Figure 14 illustrates the effect on the total claims distribution.

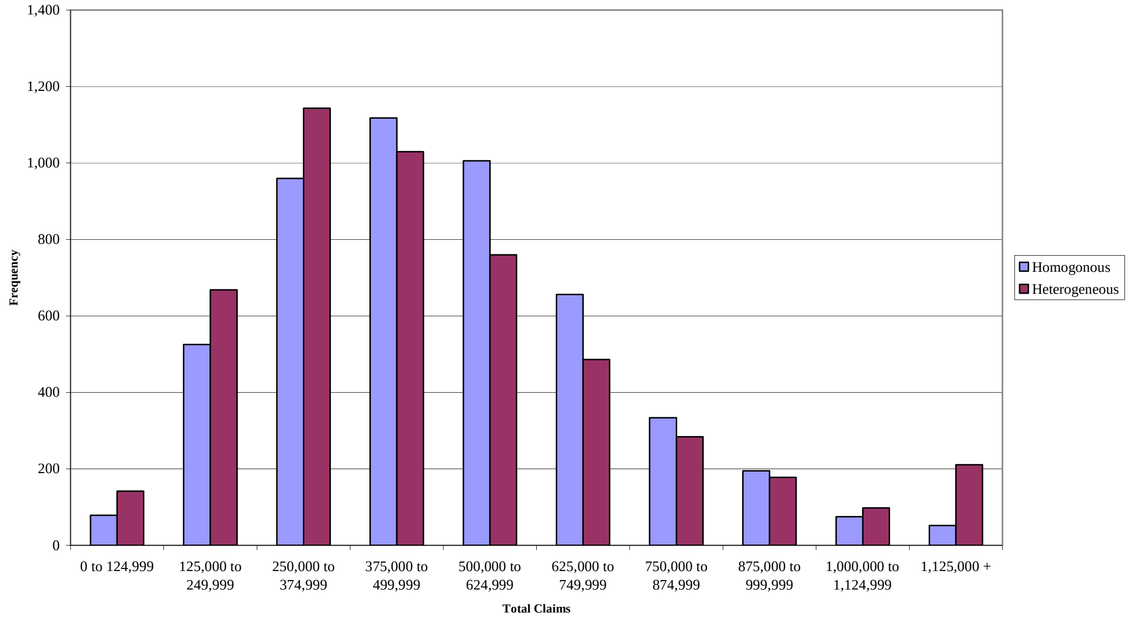

Figure 14. Frequency plot of total claims – using homogonous and heterogeneous assumptions

4 DISCUSSION

4.1 Monte Carlo Model

The model to simulate claims from a group policy is simple to implement. In addition to this it is very easy to set up a spreadsheet that can simulate claims for a group based on actual member mortality rates and sums assured. This can also be simplified to a model based on an average mortality rate and an average claim with an assumed distribution. In this case we used the exponential distribution but other distributions (discrete and continuous) are fairly simple to implement, more so where the specific inverse cumulative distribution function is readily available in the application being used or can be expressed in a formula.

The more difficult part of building a model is correctly allowing for parameter risk. It would probably require some subjective judgement to ascertain a level of parameter risk. This paper analysed the impact of parameter risk on profit shares, but it did not analyse methods to ascertain or estimate the amount of parameter risk. It may be possible to gain some measure of parameter risk from the pricing process for the underlying mortality risk. For example, the credibility rating process may provide some input in this decision. In the case analysed here the parameter risk had a fairly small impact on the rate. This may not always be the case.

4.2 Alternative uses of the model

This model is not just useful for profit share pricing but can also be used for other purposes. Other potential uses in the group risk environment are:

i) For partial self-insurance, this model can be used to assess the insurance requirements for the group. This would be done by adjusting the sums assured (S_i) for the insurance structure and adding the known cost of insurance. Using the above information the distribution of the cost of benefits (in terms of cost of insurance and retained claims) can be analysed and an optimal insurance structure obtained given a risk profile of the group.

ii) It could also be used by an insurer to assess the potential capital requirements of a group policy.

iii) The cost of stop loss covers for a group's experience can be assessed using the model. A group would need to be fairly large for this type of cover to be practical but it would be relatively easy to adjust the model to allow for pricing stop loss covers. The insurer would need to pay extra attention to the assumption of independence of risk in this case.

4.3 Profit Share Pricing

We believe that the model is useful in deriving a loading for profit shares.

Implementing this kind of process in practice can, however, be problematic. One of the main problems is that clients that have been used to getting something for nothing may be reluctant to start paying for profit shares. The layman's argument is that the insurer only pays out when experience is good and that therefore the profit share should not add to cost. The theory of profit share pricing is sound but it may be more difficult to implement when quoting to clients. However, the small margins in the group market should encourage insurers to quote the technically correct price for profit shares, as profit shares are likely to reduce the available margins (which may already be insufficient). Even where the insurer cannot actually quote the theoretically correct rate, it helps decision making because it quantifies an unknown cost and thus highlights the value of the benefit that is being given.

Our analysis also shows that it is fairly simple to derive a formula to provide an estimate for the cost of a profit share, given that the profit share follows a certain structure. This would aid quick implementation for non-technical quotes staff. It will also make it possible to easily implement this on a computer system. Judgement would, however, still be required to assess the potential for parameter risk.

The simplified formula derived to approximate profit share loadings does not have a good fit through the data. Furthermore the data used was artificial. However given that profit shares are often not priced for at all, we feel that allowing for a simple measure of the cost (if somewhat incorrect) would be better than no measure. Given a database of actual groups covered and best estimates for their mortality, this simple formula could easily be refined to fit actual data and to allow for varying values of F.

Throughout this paper we assumed that the margins in the profit share calculation and the actual margins in premiums correspond. However, this is not always the case and the profit share loading may need to be increased or reduced. Using explicit modelling this would be easy to implement. The formula would need to be adjusted to allow for this.

We have shown that the simple model has its shortcomings in cases where heterogeneity is present. South Africa generally has a population with varied mortality experience and wealth. This diversity is also evident to a greater and lesser extent in every group in the market. It is not uncommon to have a group containing both a large group of employees with low income and low levels of education (with corresponding high levels of mortality and low benefit levels) and a smaller group of management (with low levels of mortality and high benefit levels). HIV/AIDS further increases this heterogeneity and also the potential for parameter risk.

One aspect of heterogeneity that was ignored in this paper was that increased heterogeneity might also lead to increased parameter risk. As our analysis showed, heterogeneity may lead to an underestimation of the cost of the profit share. However heterogeneity may also lead to an increased probability of parameter risk, and this could therefore also impact the cost of the profit share.

Independence of risks was assumed throughout. This could be considered somewhat problematic as by definition, lives in a group are generally non-independent. Lives are usually located close together geographically (if not in the same building or complex of buildings) and may be exposed to similar risks due to being in the same geographical location and/or due to sharing exposure to the same industry. Thus independence does not hold. The impact on profit share pricing is not immediately obvious as it depends on how the risk is allowed for in the actual premiums that are charged to the client. It is however clearly an area of concern to the insurer when using a similar methodology in pricing stop loss structures.

We have not considered the impact of variation of assumptions on profit shares structured differently (for example structures similar to that in Table 2). The impact of increased variability is likely to be greater for a profit share where the scale goes up to 100%. If the level where 100% sharing starts applying were lower, the impact of increased variability would be even greater. However, profit shares of structures as per 2.3 are common in the group risk market.

5 CONCLUSION AND RECOMMENDATIONS

We feel that we have expanded on using Monte Carlo simulation in the group risk environment. We have used this model to derive a basis for estimating the cost of profit shares directly and have shown that it is possible to derive a simplified basis to estimate these costs based on expected number of deaths. We have also identified various potential shortcomings of the model that may affect the results.

We believe that some of these shortcomings could be investigated further. These include:

i) Incorporating allowance for heterogeneity in the group membership profile and experience. However, measuring the actual heterogeneity of lives in a group would be very difficult. It would require defining the potential sub-groups and monitoring their experience separately. It may not be possible to explicitly quantify some sources of heterogeneity in the data.

ii) Allowing for parameter risk objectively. This would depend on the exact pricing model being followed by the insurer and may be very difficult to quantify objectively.

The main conclusion we draw from this analysis is, however, that it is possible to relatively accurately allow for the cost of providing profit shares on a pragmatic basis.

Furthermore the analysis here has applications other than profit share pricing, specifically stop loss pricing and retention analysis. In fact it has application in any situation where the total claims cost distribution needs to be analysed.

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APPENDIX A

A1 Overview

This appendix sets out a calculation for the profit share of an actual group policy. The calculation is done using three different methods. Each method is overviewed separately below and the results are compared.

A2 The Group

This is a group of employees of a medium sized company.

Pricing for the group was based on book rates adjusted for experience. Individual member data (sum assured, age and gender) was available and this was used to calculate individual mortality rates for each member (adjusted for experience). This information was used to calculate expected number of claims and the claim amount.

Number of members	1,748
Expected Deaths	4.67
Average Mortality Rate	0.002670
Risk premium	R 2,469,188
Average Claim	R 529,137

Table 5. Group Characteristics

A profit share of 50% of profit was modelled. Profit and expense margins were assumed to be 0%.

A3 Calculation methodologies

Method 1 involved using the approximation formulae derived in 3.2 to estimate the value of l.

Method 2 used the data available in Table 5 and modelling the group assuming average mortality and exponentially distributed claim amounts.

For Method 3 individual mortality rates and sums assured were used and the total claims cost and resulting profit shares were simulated. This should be the most accurate of methods employed.

None of the methods allowed for parameter risk.

A3 Comparison of results

The loading derived by the methods are tabulated below:

Method 1	11.0%
Method 2	11.5%
Method 3	10.4%

Table 6. Estimated profit share loadings

The distributions of total claims for Method 2 and Method 3 are shown in Figure 15.

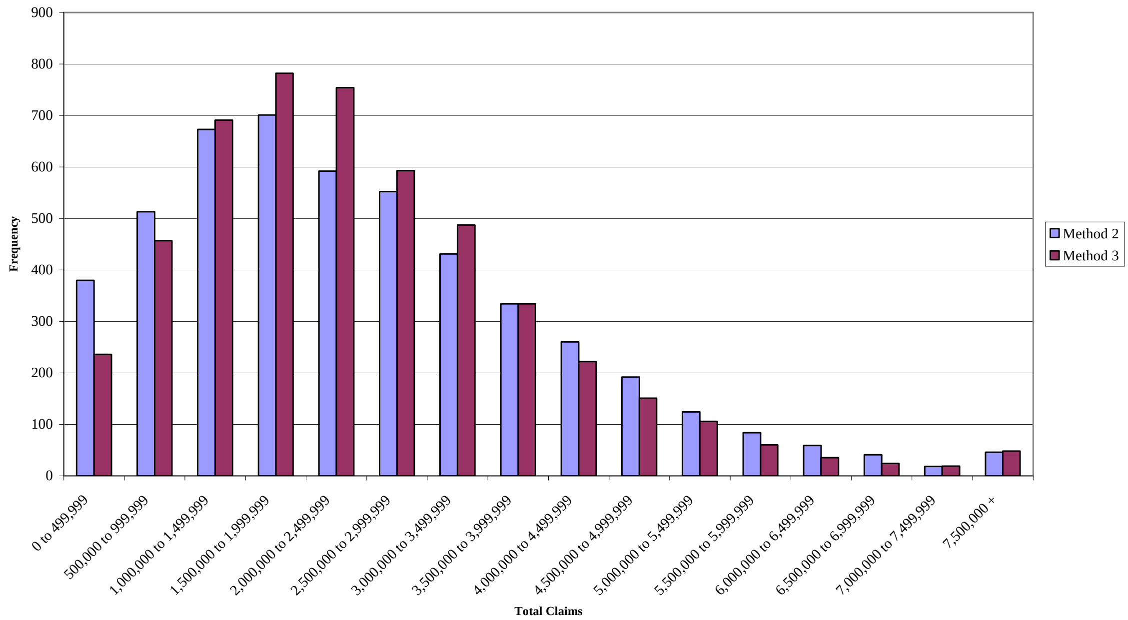

Figure 15. Frequency plot of total claims – Method 2 and Method 3

From the chart it seems that total claims under Method 2 has a greater variance than under Method 3. The standard deviation for Method 2 is 1,610,556 versus 1,511,620 for Method 3. This would explain why Method 3 produces a lower loading than Method 2.

The lower variation of total claims for Method 3 is probably due to claims amounts not being distributed exponentially. For Method 3 the individual mortality rates and sums assured effectively determine the claim amount distribution.

In this analysis we have not allowed for parameter risk. We need to allow for the uncertainty of the underlying rates. We could allow for parameter risk similarly as in 3.4 by varying the overall mortality rates. However assessing this would be fairly difficult. We should also allow for the risk that whilst the overall rates may be correct the mortality rates for different ages or genders or other categories of members may be incorrect.