# **Paper Title:** Allowing for Default Risk in Defined Benefit Pension Schemes


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## Abstract

The recent worldwide downturn in the stock markets has demonstrated that periods of weak equity markets could lead to large pension fund deficits. In the US, the total pension deficit in the pension plans for the S&P 500 companies was $164.3 billion in 2004; whilst in 1999, before the stock market downturn, the S&P 500 companies had a total pension surplus of $280 billion. Furthermore, the PBGC reported a deficit of $22.8 billion for its single-employer program in 2005; whilst the same program had a historic surplus of $9.7 billion before the downturn in 2000. This descent into deficit was due to the fact that during the fiscal years 2001 to 2003 the total number of pension plans terminated by financially-distressed companies more than doubled compared to the fiscal years 1998 to 2000. These terminations lead to a thirty-fold increase in the total net claim amounts paid by the PBGC.

In Europe, the total pension deficit in the pension plans for the Dow Jones STOXX $50^{SM}$ companies was 116 billion in 2004. The worst-affected Dow Jones STOXX $50^{SM}$ companies were those based in Germany, Spain and the UK. Similar effects have been felt by companies in the Far East, particularly in Japan.

These pension deficits have in turn lead to increased scheme contributions at a time when pension scheme sponsors have been financially weakest and hence least able to pay. As such, pension and investment actuaries are now advocating the inclusion, in the actuarial and investment modelling of pension funds, of the probability of the sponsor defaulting on the pension obligations. In this paper we assimilate the ideas of Marcus (1987), Estrella and Hirtle (1988) and Boyce and Ippolito (2002) to develop a modelling framework that explicitly allows for sponsor default risk in a pension scheme. We apply the framework to a realistic model of a final-salary defined benefit pension scheme which is assumed to be invested only in equities and bonds.

Using stochastic projections the results show that for our model pension scheme and investment model the optimal asset allocation strategy over a short-term horizon is to invest 100% in bonds. However, over medium-term and long-term horizons stochastic projections show that the pension scheme would have to follow equity-backed asset allocation strategies.



*Keywords*: Actuarial modelling, Pension deficits, Optimal asset allocation, Stock Market downturn.

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# 1 Introduction

The financial strength of sponsoring employers of defined benefit pension schemes is currently one of the most topical issues amongst pensions actuaries and investment consultants worldwide. This has stemmed from the worldwide downturn in stock markets. In the major world economies where defined benefit pension schemes are prevalent, large deficits in such schemes are now common.

In the United States, Blitzer et al. (2005) report that the total deficit for the S&P 500 companies was $164.3 billion in the year 2004 and $164.8 billion in the year 2003. For the year 2005 they predict a total deficit in the region of $140 to $150 billion. Meanwhile, in the year 1999, before the slump in stock markets, the S&P companies had a total surplus of $280 billion in their pension plans. Table 1 shows how the pension surpluses have shrunk to pension deficits for S&P 500 companies during the stock market downturn. We observe that, for all S&P 500 companies, the average funding level for pension plans has fallen from 128% in 1999 to 88% in 2004. These companies invest, on average, 65% in equities, 30% in fixed income securities and 5% in other securities.

| Year | Pension Assets $bn | Pension Liabilities $bn | Pension Surplus/ (Deficits) $bn |
|---|---|---|---|
| 1999 | 1,274.1 | 994.1 | 280.0 |
| 2000 | 1,238.9 | 1,012.9 | 226.0 |
| 2001 | 1,089.9 | 1,087.0 | 2.9 |
| 2002 | 951.0 | 1,169.5 | (218.5) |
| 2003 | 1,113.5 | 1,278.3 | (164.8) |
| 2004 | 1,265.3 | 1,429.7 | (164.3) |

Table 1: Total Pension assets, liabilities and surplus/deficits for S&P 500 companies.
Source: Standard & Poor's Historical Pension Data

Table 2 shows the pension deficits in some of the largest pension plans (by market value of assets) in the S&P 500 companies during the stock market downturn. We observe that the average pension deficit for these four pension plans was $13.656 billion in 2002, $8.202 billion in 2003 and $7.756 billion in 2004.

| | Pension deficit $bn | | |
|---|---|---|---|
| Company | 2002 | 2003 | 2004 |
| Boeing | 7.137 | 6.722 | 3.804 |
| Ford Motor Co | 15.611 | 11.689 | 12.306 |
| General Motors Corp | 25.440 | 8.644 | 7.531 |
| IBM Corp | 6.435 | 5.754 | 7.382 |

Table 2: Pension deficits for some of the largest pension plans in S&P 500 companies for years 2002 to 2004.
Source: Standard & Poor's Historical Pension Data

The stock market downturn has also hugely affected the Pension Benefit Guaranty Corporation (PBGC) in the United States. In a November 2005 press release the PBGC reported that more "...people [that] may have lost [pension] benefits promised to them by their employers..." have now been added to the PBGC's "...rolls over the past three years than in the previous 27 years..." (PBGC (2005*a*)). Furthermore, the PBGC reported a deficit of $22.8 billion in the year 2005 arising from single-employer plans. This compares to a historic surplus of $9.7 billion in the year 2000 for the single-employer program.

The PBGC's descent into deficit was caused, according to a statement made before the US Senate by the PBGC's director, "...by the failure of a significant number of large companies with highly underfunded plans..." (Kandarian (2003)). He further noted that during the 2001 and 2002 financial years the "...PBGC's surplus rapidly deteriorated and [had] disappeared altogether [by 2003], leaving the PBGC with a deficit of $3.6 billion..." The Pension Insurance Data Book for 2004 reports that from 1998 to 2000, judging by the fiscal year of termination, a total of 211 financially-distressed companies terminated their underfunded pension plans. These terminations lead to the PBGC paying a total net claim amount of $311.6 million. On the other hand, from 2001 to 2003 the total number of such terminations doubled to 427; whilst the total net claim amount paid by the PBGC increased thirty-fold to $10.6 billion (PBGC (2005*b*)).

Meanwhile, defined benefit plans for companies based in European companies also reveal large deficits. Table 3 shows the total deficits for pension plans for companies in the Dow Jones STOXX 50℠ for various European countries for the years 2003 and 2004. Thus, for example, the total deficit for pension plans of Dow Jones STOXX 50℠ companies based in Germany was €34.7 billion in 2004; whilst for Dow Jones STOXX 50℠ based in France the total deficit in the pension plans was €10.9 billion. Moreover, the total pension deficit for all Dow Jones STOXX 50℠ pension plans was €114 billion in 2003 and €116 billion in 2004.

| | Total pension deficits €m | |
|---|---|---|
| Country | Year 2003 | Year 2004 |
| Germany | 33,351 | 34,746 |
| Spain | 11,964 | 11,801 |
| UK | 39,384 | 35,873 |
| Netherlands | 11,121 | 12,717 |
| France | 10,892 | 10,913 |
| Sweden | 1,017 | 1,146 |
| Italy | 3,239 | 3,057 |
| Switzerland | 3,127 | 5,584 |
| Finland | 122 | 54 |

Table 3: Total pension deficits for Dow Jones STOXX 50℠ companies in European countries.

Source: Lane Clark & Peacock Survey

Table 4 shows the deficits for the years 2003 and 2004 of some of the largest pension plans (by market value of assets) in the Dow Jones STOXX 50℠ in various European countries. Thus, for example, the total pension deficit for DaimlerChrysler and Siemens in Germany was €10.783 billion in 2003 and €9.73 billion in 2004; whilst the total pension deficit for Banco Santander Central Hispano (BSCH) and Telefónica in Spain was €11.902 billion in 2003 and €11.707 billion in 2004; and the total pension deficit for British Petroleum (BP), British Telecom (BT) and Royal Bank of Scotland (RBS) Group in the United Kingdom was €18.786 billion in 2003 and €14.688 billion in 2004.

| Country | Company | 2003 deficit €m | 2004 deficit €m |
|---|---|---|---|
| Germany | DaimlerChrysler | 5,804 | 6,644 |
| | Siemens | 4,979 | 3,086 |
| Spain | BSCH | 9,388 | 9,639 |
| | Telefónica | 2,514 | 2,068 |
| UK | BP | 3,200 | 3,382 |
| | BT | 12,798 | 7,271 |
| | RBS Group | 2,788 | 4,035 |
| Netherlands | Phillips | 1,175 | 882 |
| | Shell group | 1,997 | 2,178 |
| | Unilever | 3,761 | 3,968 |
| France | AXA | 3,083 | 3,457 |
| | Total | 2,771 | 2,755 |
| Sweden | Ericsson | 1,017 | 1,146 |
| Italy | Telecom Italia | 1,514 | 1,356 |
| Switzerland | Credit Suisse | 830 | 1,170 |
| | UBS | 550 | 1,432 |
| Finland | Nokia | 122 | 54 |

Table 4: Pension surplus/deficits for a selection of Dow Jones STOXX 50$^{\mathrm{SM}}$ companies for European countries.
Source: Lane Clark & Peacock Survey

In the Far East, large pension plans have also been adversely affected by the downturn in stock markets. For instance, in Japan, the 2005 Annual Report for the Nippon Telegraph & Telephone (NTT) Group revealed a pension deficit of ¥104.507 billion (equivalent to $976.7 million) in 2005; whilst the pension deficit for the NTT group in 2004 was ¥106.841 billion.

Meanwhile, the annual reports from 2000 to 2005 for another Japanese company, Matsushita Electric Industrial Co Ltd, show that the total pension deficit for its pension plans grew from ¥725.4 billion in 2000 to ¥1,531.0 billion in 2003. The pension deficit in 2004 was ¥828.0 billion.

These pension deficits have lead to pension scheme sponsors being required to pay large contributions into their schemes at a time when most sponsors have also been very weak financially and hence least able to pay. Following this worldwide decline in the funding levels of defined benefit pension schemes, pension and investment actuaries are now advocating the inclusion, in the pension funding and investment decisions, of the probability that the sponsor might default on the pension obligations. Such defaults could, for example, be due to the sponsor becoming bankrupt. Orszag (2003) observes that the pension scheme's asset allocation should, among other things, depend on the 'covenant risk'.

Pension scheme sponsor bankruptcy has been an area of research for at least two decades. Most of the early studies focused on valuation of pension insurance in the light of the PBGC in the United States. Marcus (1987) considers a contingent claim approach to the valuation of PBGC insurance in a multi-period setting. He considers two models of pension insurance. Under the first model the pension scheme is terminated by the sponsor at a time which maximizes value to the sponsor. Whilst under the second model the pension scheme is terminated following the sponsor's bankruptcy. Marcus shows that under both models the sponsor would follow a "razor's edge" funding policy. That is, the pension scheme would be funded to either the maximum or the minimum level permitted. (A similar result was obtained by Bulow (1981) and Harrison and Sharpe (1982) albeit in one-period models).

For practical actuarial modelling purposes we find the approaches of Estrella and Hirtle (1988) and Boyce and Ippolito (2002) instructive. Estrella and Hirtle (1988) explicitly model the key pension fund variables (i.e. the sponsor's contribution, pension liabilities, pension assets and membership profile) and also the sponsor's company debt and assets. They suggest an empirical model of bankruptcy estimated using data from a sample of US failed and ongoing companies. Under this model a pension scheme is terminated if the probability of bankruptcy of the sponsoring company is greater than (or equal to) a pre-specified target probability of bankruptcy.

Boyce and Ippolito (2002) also suggest an empirical model of bankruptcy, albeit with more parameters than in the Estrella-Hirtle model. Under the Boyce-Ippolito model a company is declared bankrupt if the probability of bankruptcy is greater than some random number generated from the uniform distribution.

In this paper we present a pension fund modelling framework which assimilates these ideas in order to find optimal asset allocation strategies. We work on the idea that periods of weak equity markets could coincide with (or lead to) large pension fund deficits and hence increased scheme contributions at a time when the sponsor might be financially weakest. Our approach involves explicitly modelling sponsor default events and basing the pension scheme's investment decision on the probability of such events. We apply the framework to a realistic model of an ongoing defined benefit pension scheme which includes, among other features, new entrants, retirees, and benefits based on final salaries. The pension assets are assumed to be invested only in equities and bonds.

Using stochastic projections the results show that for our model pension scheme and investment model the optimal asset allocation strategy over a short-term horizon is to invest 100% in bonds. However, over medium-term and long-term horizons stochastic projections show that equity-backed asset allocation strategies are optimal.

The rest of this paper is organized as follows: in Section 2 we outline the problem of pension funding and asset allocation and the formulation of the probability of sponsor default. In Section 3 we consider the measurement of risk in the presence of default. Then in Section 4 we present a case study of our proposed framework and the main results. We conclude in Section 5 with a summary of our main findings and suggestions for further work.

# 2 Pension Fund Problem

## 2.1 Funding and Asset allocation

We formulate the risk management problem as follows. We consider a discrete-time framework where the pension fund at time $t$, $f(t)$, can be written as

$$f(t) = (1 + r(t))\ [f(t-1) + c(t-1)] - B(t) \tag{1}$$

where $r(t)$ is the total return on the pension scheme assets in the year $(t-1, t)$, $c(t-1)$ is the contribution in the year $(t-1, t)$ payable at time $t-1$, and $B(t)$ is the benefit outgo in year $(t-1, t)$ payable at time $t$.

We assume that a proportion $\alpha$ of the fund at time $t$ is invested in equities whilst $1-\alpha$ is invested in fixed-interest bonds. The fund is then annually rebalanced. Thus $r(t)$ can be written as

$$r(t) \;=\; \alpha \left( \frac{R_1(t)}{R_1(t-1)} - 1 \right) + (1-\alpha) \left( \frac{R_2(t)}{R_2(t-1)} - 1 \right) \tag{2}$$

where $R_i(t)$ is the accumulation at time $t$ of 1 invested at time 0 in the $i$th asset ($i = 1$ for equities and $i = 2$ for bonds) with all income reinvested.

We also assume that the contribution in the year $(t-1, t)$ can be written as

$$c(t-1) \;=\; c(0) + k\, D(t-1) \tag{3}$$

where $c(0)$ is the initial contribution at time 0, $k$ is the spread parameter and $D(t-1)$ is the pension fund deficit as time $t-1$ calculated as the difference between the pension fund's total discontinuance liability at time $t-1$, $L(t-1)$, and the pension fund assets at time $t-1$. That is,

$$D(t) = L(t) - f(t). \tag{4}$$

We assume that the total discontinuance liability at time $t$, $L(t)$, comprises of the discontinuance liability in respect of active members and the discontinuance liability in respect of current pensioners. The total benefit outgo at time $t$, $B(t)$, comprises of transfer values in respect of members withdrawing from the scheme, benefit payments for current pensioners, and the total payment in respect of deaths amongst active members.

We assume that stochastic projections of the pension fund are to be undertaken over a horizon of $T$ years. A major factor in these projections is the probability of a default event at the end of year $(t-1,t)$, $t \leq T$. If a default occurs at time $t$ then the scheme ceases to operate and the deficit at that time is calculated.

Default events are assumed to depend on a probability of default, $q$, in year $(t-1,t)$ and possibility of a deficit at time $t$. In Section 2.2 below we give details of how $q$ is calculated. We assume that a default event occurs if the scheme is in deficit and, following Boyce and Ippolito (2002), $q$ is less than some randomly generated number.

The decision maker's problem at time 0 is to optimally choose the asset allocation proportions $(\alpha, 1-\alpha)$ and the initial contribution $c(0)$ to minimize the expected deficit defaulted at time $T$ if default occurs during the projection period.

## 2.2 Formulation of the Probability of Default

The underlying idea in our analysis is that the state of the economy could have a significant effect on the strength of the sponsoring employer and the likelihood of honoring the scheme contributions. That is, weak financial markets could lead to large pension scheme deficits at a time when the sponsor is financially weak and thus unable to meet with large additional scheme contributions.

We thus use the movements in the financial markets as a signal of the strength of the scheme sponsor and the prospect of default. Following Cochrane (2001, p.150), who observes that the return on a broad-based portfolio can be used to measure the state of the economy, we use the return on the tangency (or market) portfolio.

We propose that the probability of default over one year, $q(r_m)$, be formulated as a logistic function of the return on the tangency portfolio. That is,

$$q(r_m) = \frac{1}{1 + p_0 + \exp\{p_1 + \alpha\,(r_m - \delta_m)\}} \tag{5}$$

where $\delta_m$ is the expected force of return on the market portfolio (i.e. $E(r_m)$); $p_0$ is a non-market component to do with risk specific to the sponsor's business sector; $\alpha$ is a risk aversion coefficient; whilst $p_1$ is necessary for model calibration purposes.

Equation (5) has two desirable properties. Firstly, the probability of default tends, in the limit, to one in 'bad' times (i.e. when the return on the market portfolio is very poor); and tends to zero in 'good' times. Secondly, when the return is equal to its expected value the probability of default is given by

$$q(\delta_m) = \frac{1}{1 + p_0 + \exp\{p_1\}}. \tag{6}$$

This property is useful for model calibration purposes.

# 3 Risk measurement in the presence of default

In this section we discuss the measurement of risk in the presence of sponsor default. We introduce the Shortfall & Default risk measure at the end of $T$ years, $SD(T)$.

Consider year $(t-1, t)$. We assume that default events can only arise at the end of the year if two conditions are satisfied: firstly, there should be a shortfall i.e. $L(t) - f(t) > 0$; and, secondly, the probability of default should be greater than some random number generated from the Uniform(0,1) distribution.

If the sponsor defaults at time $t$ then we accumulate the shortfall defaulted to the end of the projection period using the risk-free force of interest. Thus the accumulated shortfall defaulted at time $T$ is just $D(t)\, e^{(T-t)\,\delta_f}$. Then Shortfall & Default risk at the end of $T$ years, $SD(T)$, is calculated as the expected value of the accumulated amount of shortfall defaulted.

# 4 Case Study

In this section we present a case study of the framework presented in the previous sections. Our main goal is to show how the framework can be used to choose the asset allocations that minimize the default risk for various projection periods.

## 4.1 Liability Model

We consider an ongoing Pension Scheme with current pensioners. We assume that on retirement at the normal retirement age of 65 a pension benefit of one-sixtieth of final pensionable salary at the date of retirement for each year of pensionable service is payable. The pensions in payment are assumed to be increased in line with increases in the Retail Prices Index, subject to a maximum increase in any year of 5%. The accrued benefits are evaluated using the current real yield on index-linked bonds generated by the asset model.

We further assume that on withdrawal prior to the normal retirement age a deferred pension benefit of one-sixtieth of final pensionable salary at the date of withdrawal for each year of service is payable. This deferred pension is revalued in the period up to the normal retirement age. However, no benefit is provided on death in service. This model is similar to that used by Haberman et al. (2003).

## 4.2 Asset Model and Investment and Contribution Strategies

We assume that the investment returns on the main asset classes are as generated by the stochastic asset model proposed by Wilkie (1995). We use the parameter estimates and 'neutral' initial conditions as set out in Wilkie (1995). It is possible to use other asset models or parameter estimates from more recent data for the Wilkie model. However, we prefer the Wilkie model for illustration purposes as it is the most comprehensively-documented actuarial model in the public domain.

We assume that the pension scheme is initially fully funded. That is, the initial market value of pension scheme assets is equal to the initial accrued benefits. The fund is allocated in equities and bonds and annually rebalanced. We consider a range of combinations of the two asset classes from 0% equities, 100% bonds to 100% equities, 0% bonds.

We consider 17 different choices of the initial contribution rates (as a percentage of the initial total salary roll, $S(0)$): 0, 2%, 4%,...,32%. As shown in equation (3) if the scheme is fully-funded at time $t$ then the contribution rate is just the initial contribution rate. Thus in this paper we use the terms initial contribution rate and normal contribution rate interchangeably.

## 4.3 Projections

We consider stochastic projections over 1 year, 6 years and 15 years. We choose these projection periods to stand for the short-term, medium term and long-term, respectively. We carry out 20,000 simulations and at the end of every projection period the Shortfall & Default risk is calculated as explained above.

The aim of this approach is to calculate combinations of asset allocation and normal contribution rate that lead to a similar level of risk. Such combinations are depicted as 'contours' and we refer to them as *indifference curves*. Following Haberman et al (2003) we identify efficient asset allocations by considering the minimum points of these indifference curves. Such minimum points show the asset allocation strategies which lead to the lowest risk for a given choice of funding strategy.

## 4.4 Estimates for the Probability of Default

We assume that the market portfolio is composed of only equities and bonds and estimate the market portfolio over one year. For simplicity, we use the same market portfolio for all projections. A slightly different approach would be to estimate market portfolios for each projection period. This would affect the expected return on the market portfolio but would have little effect on the risk aversion coefficient $\alpha$. We leave the investigation of this approach as a problem for future work.

From the Wilkie model we estimate that the market portfolio comprises 56% equities and 44% bonds, with an expected rate of return of 10.44% and standard deviation of 0.128. The risk-free rate of interest is estimated by considering the mean nominal return on cash over one year in the Wilkie Model.

To estimate the risk aversion coefficient, $\alpha$, we consider the case of an investor with an exponential utility function and assume that asset returns are normally distributed. Cochrane (2001, p.154-155) uses these assumptions to derive the Capital Asset Pricing Model (CAPM). He shows that under this version of the CAPM we obtain a link between the coefficient of absolute risk aversion and the market price of risk (i.e. the price of systematic risk):

$$\alpha\,\sigma^2(r_m) = E(r_m) - \delta_f \tag{7}$$

where $\sigma^2(r_m)$ is the variance of the force of return on the market portfolio, $\delta_f$ is the risk-free force of return; and $E(r_m) - \delta_f$ is the price of systematic risk.

Thus using equation (7) we obtain an estimate value for $\alpha$ of approximately 2.49.

To estimate $p_1$ we pre-set the probability of default when the force of return on the market portfolio is equal to its expected value and then use equation (6). Thus we need assumptions concerning $p_0$ and $q(\delta_m)$.

For simplicity, in this paper we assume that $p_0$ is zero. We set $q(\delta_m)$ to be equal to 0.02. This value is inferred from Crosbie and Bohn (2002) who, in their analysis of default on general company debt, observe that "...[t]he typical firm has a default probability of around 2% [0.02] in any year...". Hence we get $p_1$ approximately equal to 3.89.

These approximations imply that if the market rises by 10% as would be expected (given the above estimates) then the probability of default is approximately 0.02 and if the market falls by 20% then the probability of default is approximately 0.04. Meanwhile, if the market falls by 50% then the probability of default is approximately 0.13. And, in the highly unlikely event of the market falling by 80%, then the probability of default is approximately 0.60.

## 4.5 Results for the Stochastic Projections

In this section we present the results for the Shortfall & Default risk for the three projection periods. For each projection period we calculate the default risk at the end of the period for various combinations of asset allocation and normal contribution rate. We present the results using indifference curves which show combinations which lead to the same level of risk.

### 4.5.1 Short-term Projections

Figure 1 shows the Shortfall & Default $SD(1)$ risk indifference curves at the end of one year. We use only four curves for illustration purposes. Each curve passes through combinations of normal contribution rate and asset allocation that lead to the same level of default risk. Thus, for example, the top curve shows all combinations that lead to a risk level of 0.0009 (expressed as a fraction of the initial discontinuance liability). Thus we would be indifferent to the combinations along a given curve.

However, by considering a fixed funding strategy, we observe that the risk level decreases as we reduce the allocation in equities. For example, consider the 15% normal contribution rate. At 60% allocation in equities the risk level is 0.0012 whilst if we reduce the allocation in equities to 20% the risk level decreases to 0.0009. In general we observe that for all funding strategies the lowest risk level occurs at 0% allocation in equities.

This implies that under the Shortfall & Default risk, for projections over one year, the optimal asset allocation is 100% bonds (and 0% equities). This is the asset allocation that, for short-term projections, minimizes the risk of sponsor default.

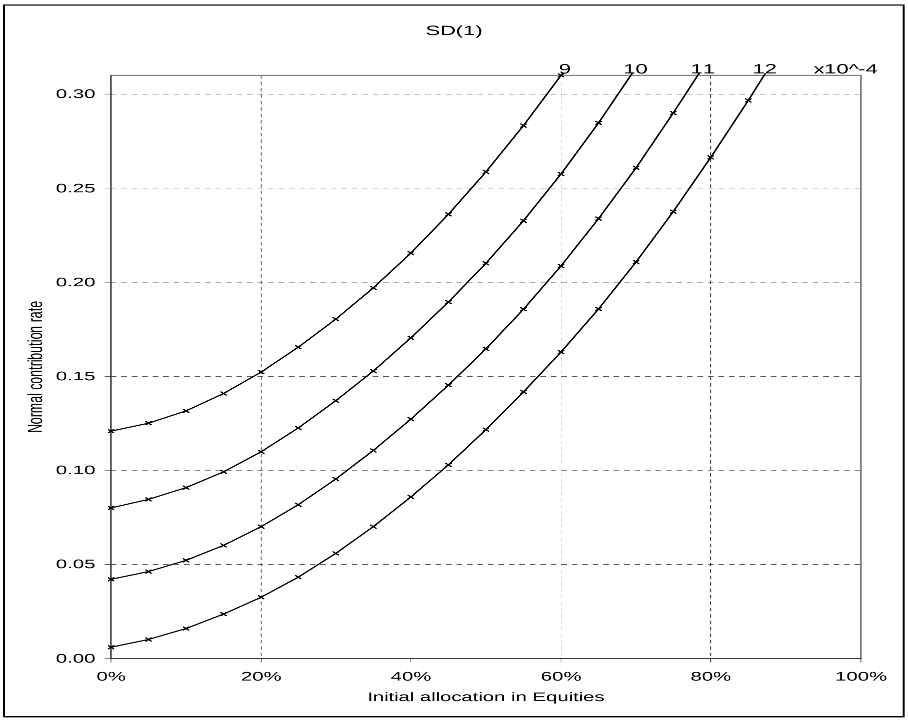


Figure 1: Shortfall & Default Risk Levels At The End Of 1 Year

### 4.5.2 Medium- and Long-term Projections

Figure 2 shows the Shortfall & Default risk levels at the end of six-year and 15-year projection periods. As for the short-term projections, the curves in Figure 2 pass through all combinations of normal contribution rate and initial allocation in equities that lead to the same level of default risk at the end of a given projection period. Thus, for instance, the top curve for projections over 6 years shows all combinations that lead to a risk level of 0.007 (as a fraction of the initial discontinuance liability). Whilst the top curve for projections over 15 years shows all combinations that lead to a risk level of 0.025.

We establish the optimal asset allocations by considering the minimum points of the indifference curves. In Figure 2 we have drawn the lines PQ to (approximately) pass through the minimum points. We observe that for a given funding strategy all asset allocations not on PQ lead to higher risk levels. Thus to the right of PQ we reduce risk by decreasing allocation in equities whilst to the left of PQ we reduce risk by increasing allocation in equities.

Hence for a given projection period the optimal asset allocations lie along the line PQ. Thus we observe that the optimal asset allocations for projections over 6 years are in the region of 60% bonds (and 40% equities). Meanwhile, the optimal asset allocations for longer term projections lie in the region of 40% bonds and 60% equities.

Thus, compared to the short-term projections, we do not get a full weighting in bonds. This is due to a combination of different factors. However, the most important factor is that better performance by equities would lead to lower average deficits in the pension scheme. This would thus imply that we get fewer default events and hence lower Shortfall & Default risk. The full effect of this would not be appreciated for the one-year projection period due to the short time horizon. However, for medium and longer term projections there would be sufficient time for a reduction in the average deficits. Thus equities would be seen as an optimal asset class for the pension fund for such projection horizons.

# 5 Conclusion

Following recent worldwide downturns in equity markets which have led to large pension fund deficits and increased scheme contributions at a time when sponsors have been least able to pay, pension and investment actuaries have been advocating the inclusion of default risk in pension funding and investment decisions. In this paper we have presented a framework showing how probabilities of sponsor default can be included in stochastic projections of a defined benefit pension scheme. Under this framework we have introduced the Shortfall & Default risk measure which calculates the expected amount of shortfall defaulted during the projection period. Using indifference curve analysis we have shown how to optimally choose the investment strategy, for any given funding strategy, in order

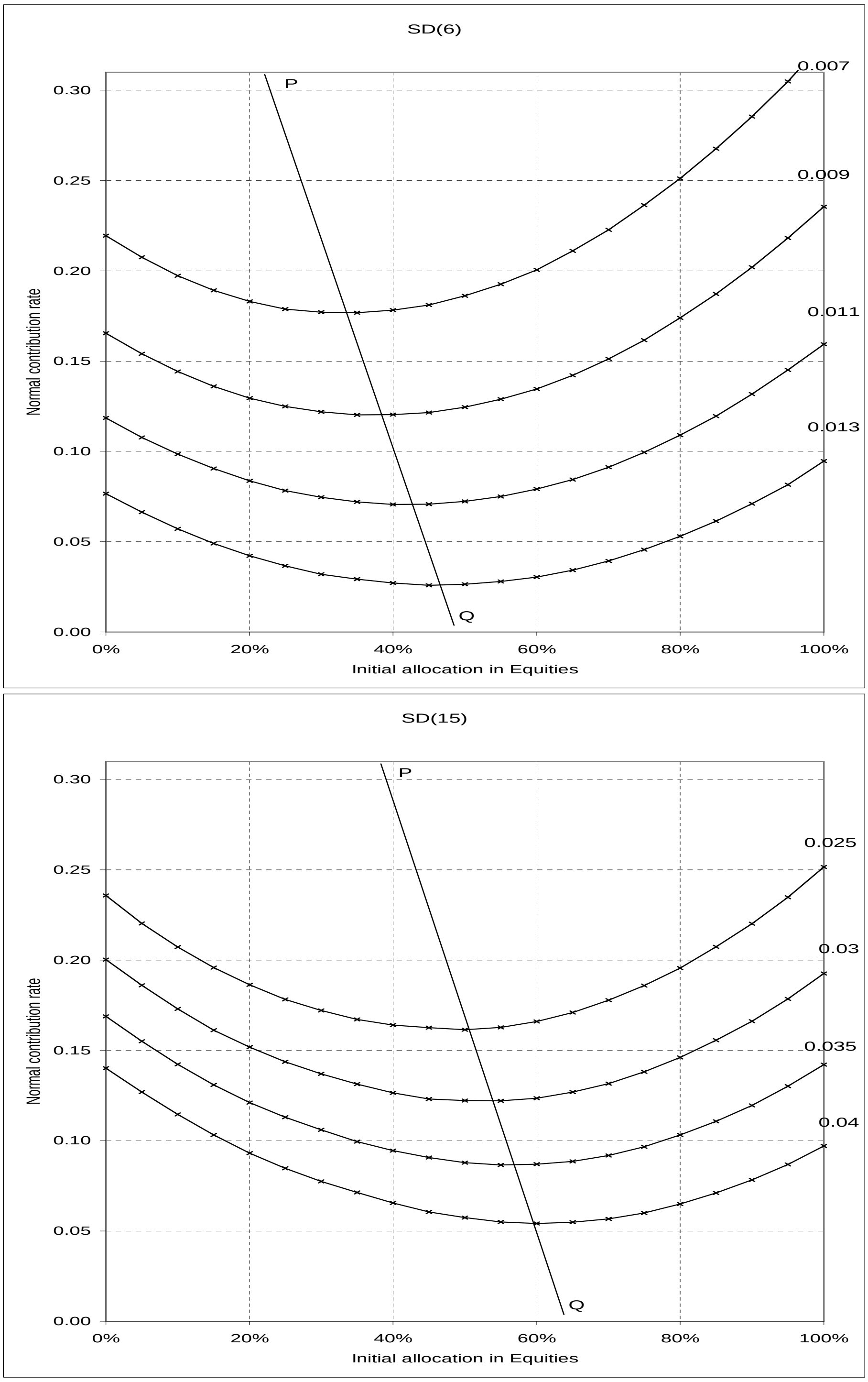


Figure 2: Shortfall & Default Risk Levels At The End Of 6 and 15 Years

to minimize default risk.

In a Case study we have shown that for short-term projections the inclusion of default probabilities leads to an optimal asset allocation of 100% bonds. However, in the case of medium-term and long-term projection periods the Case study shows that the inclusion of probabilities of default does not lead to a full (100%) bond investment as in the short-term case. On average we allocate approximately 40% equities and 60% bonds in the medium-term projections; whilst for the long-term projections we allocate 50% equities and 50% bonds.

In the short-term, our results confirm the argument that the inclusion of probabilities of default could lead to bond-only optimal asset allocations. However, in the long-term equity-backed asset allocations are optimal regardless of whether or not probabilities of default are included in the stochastic projections.

In future we hope to extend this work in several ways. The results we have obtained depend on the investment model we have used. Hence we hope to apply this framework to a different investment model. A change in the liability model could also be considered. For example, closing the scheme to new entrants or using a career-average salary instead of final salary.

# References


Blitzer, D. M., Silverblatt, H. and Guarino, D. (2005), S&P 500 2004 Pension Status Report, Annual report, Standard & Poor's.

Boyce, S. and Ippolito, R. A. (2002), 'The Cost of Pension Insurance', *The Journal of Risk and Insurance* **69**, 121 – 170.

Bulow, J. I. (1981), Pension Funding and Investment Policy, Mimeo, Stanford University.

Cochrane, J. H. (2001), *Asset Pricing*, Princeton University Press.

Crosbie, P. J. and Bohn, J. R. (2002), Modeling Default Risk, Mimeo, KMV, LLC.

Estrella, A. and Hirtle, B. (1988), 'Estimating the Funding Gap of the Pension Benefit Guaranty Corporation', *Federal Reserve Bank of New York Quarterly Review* **Autumn**, 45 – 59.

Haberman, S., Day, C., Fogarty, D., Khorasanee, M. Z., McWhirter, M., Nash, N., Ngwira, B., Wright, I. D. and Yakoubov, Y. (2003), 'A Stochastic Approach to Risk Management and Decision-Making in Defined Benefit Pension Schemes (with discussion)', *British Actuarial Journal* **9**(3), 493 – 618.

Harrison, J. and Sharpe, W. F. (1982), Funding and Asset Allocation for Defined-Benefit Pension Plans, Working Paper No. 935, NBER.

Kandarian, S. (2003), Statement before the United States Senate Committee on Finance, Technical report, March 11, 2003.

Lane Clark & Peacock (2005), Accounting for Pensions UK and Europe, Annual survey 2005, Lane Clark & Peacock Actuaries & Consultants.

Marcus, A. J. (1987), Corporate Pension Policy and the Value of PBGC Insurance, in: Bodie, Z., Shoven, J. and Wise, D. eds., *Issues in Pension Economics*, Chicago: University of Chicago Press.

Orszag, M. (2003), 'Employer pensions, risk and asset allocation', *Rome, Italy* .

PBGC (2005*a*), 'PBGC Public Affairs', *Press Release November 15, 2005* .

PBGC (2005*b*), *Pension Insurance Data Book*, Pension Benefit Guaranty Corporation, Number 9, Spring 2005.

Wilkie, A. D. (1995), 'More on a Stochastic Asset Model for Actuarial Use', *British Actuarial Journal* **1**, 777 – 964.